Deaths from radiation-induced cancer amongst the Hiroshima and Nagasaki bomb survivors

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It is now possible, as the 80th anniversary of the atomic bombing of Hiroshima and Nagasaki approaches, to estimate, with good accuracy, the long term effect of the flash of radiation released by the two bombs in early August 1945. While, in the past, there have been fears that delayed effects would very seriously impair the lives of all those who survived, it is clear, now that most of the evidence is in, that deaths of survivors caused by radiation will make up only one or, at most, two per cent, of the total number. Industrial pollution may provide the closest analogy to the damage to human health the radiation has caused in the long term. Survivors will have faced an additional chance of death that is between 20 and 40 per cent of the probability of dying from man-made air pollution in the UK. The long delay between exposure and possible death means that even those who die prematurely will probably have lived a long life. Even survivors receiving an average dose of 2,250 mGy, far above the threshold for acute radiation sickness, can expect to live to a median age of 78 years 169 days, nine months more than the life expectancy at birth for an infant born in the UK in 2000, and a year and ten months longer than an American baby born in that year could expect to live. The average survivor can expect to lose about 1½ months of life compared with an inhabitant of one of the two cities who received a negligible dose. The survivors of the atomic bombs have shared in the general uplift in Japan's life expectancy since the end of the Second World War. Despite the slightly elevated risk the survivors have faced as a result of their exposure to radiation in 1945, their lives will, on average, have been decades longer than those of their forebears. While the Life Span Study (LSS) carried out by the Radiation Effects Research Foundation (RERF) starts on 1 October 1950, the time of a national census, this paper uses actuarial methods to explore the period from August 1945 to 1 October 1950 as well. This is allowed by introducing the notion of an LSS Core Precursor Cohort, which is a superset of the LSS Core Cohort used in the RERF studies. Probabilistic modelling, common in reliability engineering, is then applied to assess the number of deaths from radiation-induced cancer, both solid cancer and leukaemia, there will be amongst the members of the LSS Core Precursor Cohort in the 110 year period between the dropping of the bombs and 2055, when the youngest possible survivor will be 110 years old. The results for this cohort are then extended to the whole population of survivors, including the military survivors who returned to their homes across Japan after the war.

1. Introduction

The long time interval, approaching eighty years, since the world's first atomic bombs fell on Hiroshima and Nagasaki allows a good estimate now to be made of the long-term health effects of the flash of nuclear radiation to which the surviving inhabitants of those two cities were exposed when the bombs exploded.

The weapons were of unprecedented power. The Hiroshima bomb fissioned enriched uranium 600 m above the ground on 6 August 1945 to produce an effect similar to 16 kt of TNT exploding. A plutonium bomb was detonated 500 m above Nagasaki three days later, unleashing the same power as 21 kt of high explosive (Jordan, 2016). Large areas of the two cities were devastated and around 200,000 people were killed immediately or shortly afterwards. Most of the deaths were from blast and burn injuries, but about 20,000 died from acute radiation poisoning within days or weeks.

The knowledge that such enormous destruction could be wrought by just two devices, of which there might be more, led the Japanese government to signal its

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Journal of Biological Physics and Chemistry **25** (2025) 8–45
Received 24 March 2025; *accepted* 12 May 2025

doi: 10.4024/03TH25A.jbpc.25.02

willingness to negotiate for peace the day after the second bomb dropped. Japan agreed to surrender unconditionally four days later.[1]

Those who survived the bombings had received an instantaneous dose of ionizing radiation of a size that depended principally on their distance from the hypocentre but also on how well they were shielded by buildings and walls. Evidence of elevated rates of cancer in both cities began to appear in the late 1940s and the 1950s, and this led to the establishment of a joint US–Japan project, the Life Span Study (LSS), to carry out medical monitoring of a large cohort, about 100,000 strong, of those who had survived the bombings and were still living in one of the cities on 1 October 1950, the date of Japan's 1950 census. The LSS project, which continues to the present day under the auspices of the Radiation Effects Research Foundation (RERF), has shown that nuclear radiation increases the risk of dying from cancer.

The epidemiological results have, usually, been framed in terms of the hazard posed by a specified dose of radiation. The approach has been of immense value in quantifying the risk faced by workers employed at nuclear power and fuel plants from radiation exposure. The knowledge has proved foundational to the setting of radiation exposure standards in the industry.

However, as Bertrand (2016) has observed, neither the general public nor, indeed, many scientists working in other fields appear to have grasped how relatively few the extra cancers at Hiroshima and Nagasaki have been. There is little doubt that fears over the residual cancer risks after the Hiroshima and Nagasaki bombs have morphed, in the minds of many, into worries about nuclear energy. This has resulted in a sizeable minority regarding the generation of electricity by nuclear power plant as too dangerous to contemplate.

The epidemiological studies at Hiroshima and Nagasaki have focused on the health of 120,321 people, who were selected from 195,000 survivors who still resided in one of the two cities on 1 October 1950. The group of one hundred and twenty thousand survivors was divided into the following four categories (Ozasa, 2018):

- Category 1: all those who were still alive and still resident in one of the two cities on 1 October 1950, and who had been within 2 km of the blast at either Hiroshima or Nagasaki. There were 34,363 people in this category on 1 October 1950;
- Category 2: all those survivors resident in one of the two cities on 1 October 1950 who had been between 2 km and 2.5 km from the hypocentre at the time of the bombing at either city (19,959);
- Category 3: a sample of those alive and resident in one of the two cities on 1 October 1950 who had been between 2.5 km and 10 km away from the hypocentre, with the sample matched, in terms of age and sex, to the survivors in Category 1 (39,419);
- Category 4: a sample of survivors resident in one of the two cities on 1 October 1950, who normally lived in either Hiroshima or Nagasaki, but who were not in the city at the time the bombs dropped, with the sample matched, by age and sex, to the survivors in Category 1 (26,580).

These survivors, all still residing in one of the two cities on 1 October 1950 and numbering 120,321 at that time, may be termed the LSS Full Group.

A total of 284,000 survivors were listed in the Japan's national census on 1 October 1950, but it was reported that 89,000 of these were living away from the two cities, and follow up was deemed impractical (Beebe et al., 1962, Jablon et al., 1964). Thousands of troops were present in Hiroshima at the time of the bombing, but they had dispersed back to the rest of Japan by 1950, and no attempt was made to trace them (Jablon et al., 1964).

A group of 86,611 survivors within the LSS Full Group, named here the "LSS Core Cohort", is of particular significance, since these were people for whom good estimates of radiation dose could be made and who: (i) had been present in Hiroshima or Nagasaki at the time of the bombings, and (ii) were still living in one of the cities on 1 October 1950. See Table 1 of Ozasa et al. (2012).

The total number of survivors in Categories 1, 2 and 3 above, namely 34,363 + 19,959 + 39,419 = 93,741 on the date of Japan's 1950 census, is greater than the number, 86,611,[2] in the LSS Core Cohort at that time. This is because individual dose information is a requirement for LSS Core Cohort membership and adequate dose estimates are unavailable for about 7,000 of the people in the first three categories. The effect on the numbers of survivors within Categories 1, 2 and 3 in the LSS Core Cohort is explained in §4.1.

The LSS Core Cohort contains the vast majority, at least 92 per cent, of those who were still resident in one

[1] Imperial War Museums, 2025, The Atomic Bombs that ended the Second World War, https://www.iwm.org.uk/history/the-atomic-bombs-that-ended-the-second-world-war (accessed January 2025).

[2] All figures, with the exception of the 120,321 LSS total, are subject to small modifications as the results of further epidemiological research come in. For example, Ozasa et al. (2019) give the 2019 figure for the LSS Core Cohort as 86,711, an increase of 0.06%. However, the figure of 86,611 will be used in this paper for consistency with the key papers that are analysed here in detail (Ozasa et al., 2012 and Richardson et al., 2009).

of the cities on 1 October 1950, who would have been within 2.5 km of the hypocentre at either city and who would have been subject to the highest doses (Ozasa et al., 2012, 2018, 2019). The observational data have been refined over the years and are of high quality.

But, despite the large number of excellent statistical studies, it is still not easy to find a clear answer to the simple but important question: How many cancer deaths did the Hiroshima and Nagasaki bombs cause in the long term? This paper, written eight decades after the events of August 1945, will attempt an answer.

A number of problems need to be addressed on the way. The LSS Core Cohort was assembled only from those who were still living in one of the two cities at the time of Japan’s 1950 census. One effect has been that the major RERF studies leave the 5.15 year interval between the time the bombs fell and the census date, namely 1 October 1950, unexplored. Moreover, while the LSS Core Cohort is a sample taken from the 195,000 people who were still resident in the cities on 1 October 1950, it is known that tens of thousands of survivors, including a large military contingent, moved away from the cities after the war and were no longer resident there when the census was carried out. Ways of extending the RERF results back in time and across different survivor populations need to be developed.

Accordingly, this paper will put the LSS Core Cohort into context through introducing the concept of an “LSS Core Precursor Cohort”, which will be a group of survivors selected using the same criteria as the LSS Core Cohort, but chosen immediately after the bombings in August 1945 rather than on 1 October 1950. An LSS Core Precursor Cohort will have the property that it will degenerate to the LSS Core Cohort of 86,611 irradiated survivors by 1 October 1950.

Past observations have shown that very few survivors succumbed to the long-term effects of radiation before the end of 1950, which means that the number of people in the precursor cohort will decrease over time in the five years after the bombings almost entirely through the normal processes of mortality and not because of radiation. The number of survivors in the precursor cohort in earlier years may, therefore, be found from working back from the LSS Core Cohort on 1 October 1950 using actuarial methods and the appropriate life tables.

Official life tables for Japan are available for the years 1947, 1948, 1949 and 1950, and approximate life tables will be constructed to cover the years 1945 and 1946.

Publicly available data will be used to estimate the number of radiation-induced fatal cancers the survivors would have experienced in the 58 years since the atomic bombs were dropped in 1945 to the end of 2003. These core figures, plus further published observations on early radiation-induced leukaemia deaths, will form the basis for predictions of the death toll over all time from the long-term effects of radiation at Hiroshima and Nagasaki.

1.1 Organization of the paper

§1 has outlined the destructive power of the atom bombs that fell on Hiroshima and Nagasaki in early August 1945, and has provided historical context for the extensive and continuing research into the long term effects of the radiation released. It has explained how the LSS Full Group was selected from the subset of survivors who were still resident in one of the two cities on the date of Japan’s 1950 census, 1 October of that year, and listed the distance categories adopted within the LSS Full Group. The criteria for selection of the “LSS Core Cohort”, which is a subset of the LSS Full Group, have been discussed in terms of the distance categories. The present study’s purpose, to estimate the total number of cancer deaths over all time amongst all the atomic bomb survivors, has been explained, as have the difficulties that need to be overcome. The development of an LSS Core Precursor Cohort has been highlighted as a key first step.

The rest of the paper is laid out as follows. §2 discusses the immediate effects of the atomic bombs and the numbers of survivors. §3 introduces the concept of an LSS Core Precursor Cohort. §4 explains the reduction in radiation dose with distance from the hypocentre, and uses the relationship between dose and distance to refine the estimated numbers in the various distance categories within the LSS Core Cohort. §5 estimates the number of deaths from radiation-induced leukaemia there would have been in the LSS Core Precursor Cohort in the period between August 1945 and 1 October 1950, then calculates the likely number of radiation-induced leukaemia deaths over all time in the LSS Core Precursor Cohort. §6 extends these leukaemia estimates to all the survivor populations. §7 presents calculations of the number of deaths from radiation-induced solid cancer over all time in the LSS Core Precursor Cohort. §8 extends the results for radiation-induced solid cancers to all survivors. §9 presents results for all radiation-induced cancers, solid cancer and leukaemia, broken down by survivor cohort. §10 discusses noncancer mortality amongst the survivors at the two cities. §11 uses prior work on survivor life expectancy, taking into account both cancers and noncancer diseases, to find the likely change in life expectancy amongst all survivors. Suicide risk amongst survivors is discussed in §12. Possible genetic damage and possible ill-health amongst children born to survivors are considered in §13. §14 contains a discussion of the results and their implications. Conclusions are given in §15.

The appendices provide more detailed information, including mathematical results. Appendix A explains how the official life tables for Japan for 1947, 1948, 1949 and 1950, taken together with official comments, form the basis from which approximate life tables for 1945 and 1946 may be constructed. Appendix B describes how survival probabilities are used in the analysis. Appendix C outlines how the age structure of the LSS Core Cohort on 1 October 1950 may be deduced from the ages that the members of the cohort would have had at the time when the bombs were dropped. Appendix D describes how the ages of the members of an LSS Core Precursor Cohort immediately after the bombings may be deduced, and the total number of members at that time calculated. Appendix E derives properties of demographically similar survivor populations. Appendix F discusses the deaths from radiation-induced solid cancer and leukaemia amongst the few members of the LSS Core Cohort who were more than 2.5 km from the hypocentre but who, nevertheless, received a dose higher than 5 mGy. Appendix G discusses how to estimate the number of deaths from radiation-induced leukaemia over all time, while Appendix H extends the treatment to deaths from radiation-induced solid cancer. Appendix I details how the number of deaths from radiation-induced leukaemia in all survivor populations are calculated for different time intervals. Appendix J fulfils a similar function for deaths from radiation-induced solid cancer.

2. The immediate effects of the atomic bombs and the numbers of survivors

It is estimated that 140,000 people (Ozasa et al., 2018) were killed outright, or died before the end of the year after the first atomic bomb was exploded over Hiroshima on 6 August 1945. The second atomic bomb, which was detonated above Nagasaki on 9 August 1945, killed 74,000 people (Ozasa et al., 2018) either immediately or within a few months.

The population of Nagasaki at the time is given as 195,000 by Britannica,[3] an figure that is confirmed by French et al. (2018), who used rice ration entitlements in July 1945 to come up with a population of 195,290. Taking the Britannica figure as correct and subtracting the 74,000 immediate death toll suggests there would have been 121,000 immediate survivors at Nagasaki.

The population of Hiroshima at the time of the bombing is subject to greater uncertainty. French et al. (2018) arrived at a figure of 255,260, based on rice ration data for June 1945, while Britannica states the population as 343,000.[4] The latter figure tallies with the statement on the US Department of Energy's site: *The Manhattan Project: an interactive history* that: "Hiroshima had a civilian population of almost 300,000 and was an important military center, containing about 43,000 soldiers".[5] The US DOE's two figures will be adopted in this paper.

Distributing the 140,000 early deaths at Hiroshima *pro rata* to the numbers of civilian and military inhabitants leads to estimates of 122,450 near-immediate civilian deaths and 17,550 near-immediate deaths amongst the Japanese troops stationed at Hiroshima. The corresponding numbers of survivors at Hiroshima are 177,550 civilians and 25,450 military, summing to 203,000. The combined numbers of survivors at the two cities, Hiroshima and Nagasaki, are then 298,550 civilian and 25,450 military on 7 August 1945, summing to 324,000 survivors in total.

The numbers of survivors would have fallen, as a result of normal mortality, as explained in §2.3, to 274,278 civilian and 24,269 military by 1 October 1950,[6] making 298,547 in total. This may be compared with the figure, reported by Beebe et al. (1962), of 284,000 survivors from Hiroshima and Nagasaki at the time of the 1950 National Census, of whom only 195,000 were still resident in one of the two cities. The widespread dispersal of the troops after demobilization and the general problem of keeping track of the large number of people who migrated to other parts of Japan after the war ended, when added to uncertainty in the starting

[3] Britannica, Atomic bombings of Hiroshima and Nagasaki, https://www.britannica.com/event/atomic-bombings-of-Hiroshima-and-Nagasaki (accessed August 2024).

[4] Britannica (op. cit.) gives Hiroshima's population as 343,000 and Nagasaki's population as 195,000 before the bombs were dropped, which gives a total population of 538,000. Folley et al., 1952, give populations of 285,712 for Hiroshima and 241,805 for Nagasaki, summing to 527,517. The numbers used by Folley et al. appear to be taken from Japan's census of 1 October 1950, and reflect the fact that Japan's population grew rapidly after the end of World War II. Hiroshima's population was recovering rapidly after the bombing, while Nagasaki's already exceeded the level it had before the atomic bomb was dropped.

[5] US Department of Energy—Office of History and Heritage Resources, 2025, *The Manhattan Project: an interactive history, The Atomic Bombing of Hiroshima*, https://www.osti.gov/opennet/manhattan-project-history/Events/1945/hiroshima.htm (accessed February 2025).

[6] The troops stationed at Hiroshima on 7 August 1945 would likely have been aged between 19 and 44 (US War Department, 1944), and a uniform distribution of ages was assumed on that date. The calculations to find the number still living on 1 October 1945 after normal mortality had taken its toll were carried out using the EYE–Gompertz life tables derived in Appendix A for the two sexes combined for 1945 and 1946, and the official Japan life tables for males for 1947, 1948, 1949 and 1950.

populations, may well explain the discrepancy, relatively small at about 5%, between Beebe's figure of 284,000 and the number, 298,547, found after allowing for normal mortality between August 1945 and October 1950. The larger figure will be retained for consistency and also conservatism in the calculation of deaths from radiation-induced leukaemia and solid cancer—slightly higher figures will result.

Based on the estimates above, about 40 per cent of the population of the two cities would have been killed at the time of the bombings or soon after, but roughly 60 per cent would have survived the atomic blasts. The US Department of War published the results of an official investigation of the bombings of Hiroshima and Nagasaki, on June 30, 1946. The US report[7] contained similar estimates to those presented above of the total number of fatalities (135,000 at Hiroshima and 64,000 at Nagasaki) and broke the causes of deaths down into three categories:

- Burns, including flash burns resulting from radiant heat: 60% of those killed at Hiroshima; 80% in Nagasaki.
- Falling debris and flying glass: 30% in Hiroshima; 14% in Nagasaki.
- Radiation: 10% in Hiroshima; 6% in Nagasaki.

Some survivors experienced burns from the flash of intense thermal radiation accompanying the explosions. While the majority of these burns were of first or second degree and healed within a month, some people received deep second degree or else third degree burns. Their recovery was impeded by a lack of expert medical care and a consequent high incidence of local infection. The more severe burns could take several months to heal and would have left extensive scarring.

Block and Suzuki (1948) found the lesions after the atomic bombings to be similar to those seen on a 19-year-old Japanese male who had sustained burns when stored petrol exploded near to him as a consequence of the incendiary bombs dropped on Tokyo on 13 April 1945. Those authors concluded that the skin scarring, which some survivors at Hiroshima and Nagasaki suffered, represented "no peculiar effect of the atomic bomb explosion".

About 20,000 people who received very high doses of nuclear radiation died from acute radiation poisoning shortly afterwards. Others, in contrast, including a good many who had received radiation doses (more than 500 mGy) that were high enough to cause acute radiation syndrome, recovered. These people will have led normal and often very long lives, but will have faced an elevated risk of contracting cancer later on, either a solid cancer or a leukaemia.

Almost all the radiation exposure occurred at the time the bombs detonated. The fact that the explosions occurred well above the ground, at a height of 500 m at Nagasaki and 600 m at Hiroshima, reduced the neutron activation of the soil, so that the radiation did not persist in the way seen at US nuclear test sites in Nevada for example.[8] Meanwhile the fireballs that were generated by the nuclear reactions swept the fission products upwards into the stratosphere and away from the target city. While some fallout dropped back to earth with rain when it cooled (given the name "black rain" because of its high soot content)[9] nuclear contamination at Hiroshima and Nagasaki was, fortunately, small. Arakawa (1962) concluded that "after the detonations in Hiroshima and Nagasaki radiation levels were such that very few individuals, if any, received significant amounts of residual radiation from external sources". It follows that the long-term health effects at Hiroshima and Nagasaki came entirely, or nearly so, from the burst of nuclear radiation released at the time of detonation, which consisted primarily of gamma rays but with a small component of neutron radiation (Cologne and Preston, 2000).

3. The LSS Core Precursor Cohort

3.1 Preliminary consideration of the number of deaths from radiation-induced cancers before 1 October 1950

Leukaemia, which is a rare condition, was the first disease to show an anomalous increase after the bombings. The number of leukaemia deaths attributable to radiation in the years before the 1950 census, the start date for the LSS study, were nevertheless low. This study will confirm (§5.2) that the number of extra deaths from 1945 to 1950 are measured in tens only.

The shortest time between radiation exposure and death for a solid cancer is generally accepted to be at least five years, and more likely ten (Richardson and Ashmore, 2005; Ozasa et al., 2018, Fig. 4, note; Marshall et al., 1983). No extra deaths from solid cancers were, in fact, observed in the interval to 1 October 1950.

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[7] Britannica, 2024, Atomic bombings of Hiroshima and Nagasaki https://www.britannica.com/event/atomic-bombings-of-Hiroshima-and-Nagasaki (accessed August 2024).

[8] Radiation Effects Research Foundation, 2025, Frequently asked questions about the atomic-bomb survivor research programmed, available at https://rerf.or.jp/en/faq (accessed January 2025).

[9] Oak Ridge Associated Universities, Black rain, https://www.orau.org/health-physics-museum/collection/nuclear-weapons/hiroshima/black-rain.html#:~:text=The%20ash%20had%20the%20effect,cases%2C%20severe%20radiation%20burns%20resulted (accessed January 2025).

The figure for deaths from normal causes will thus outweigh, by orders of magnitude, the number of deaths from radiation-induced cancers, whether solid or leukaemia. Little accuracy will be sacrificed, therefore, by assuming that all deaths in the roughly five year interval to the autumn of 1950 will have been the result of normal mortality. This allows life tables for Japan as a whole to be used to back-calculate, from the number, 86,611, of members of the LSS Core Cohort on 1 October 1950, how many survivors there would have been in an LSS Core Precursor Cohort at any earlier time, going right back to the time the bombs exploded.

3.2 Representing the time of the bombings by a single, central date

The bombs were dropped close to each other in time, 6 August 1945 at Hiroshima and 9 August 1945 at Nagasaki, which allows a notional, central value to be used for simplicity. 7 August 1945 was selected as a weighted average of the dates of the two bombings. It was chosen to lie closer to the time of the Hiroshima bomb because of the roughly 2:1 ratio of survivors from the two cities in the LSS Core Cohort: 58,494 from Hiroshima and 28,117 from Nagasaki on 1 October 1950 (Ozasa et al., 2012, Table 1).

The single-date approximation is justified by the long period between induction and death for all radiation-induced cancers, including leukaemia: years or, more usually, decades. Thus the difference of a day or two between the notional, central date and the actual dates the bombs fell will have a negligible $n_{RS}(t_{1945})$ effect on accuracy.

3.3 The number of survivors in an LSS Core Precursor Cohort immediately after the bombings

The selection criteria ensure that the LSS Core Cohort is made up exclusively of civilians, although a sizeable number of Japanese troops are known to have been stationed at Hiroshima at the time the bomb fell. A good many soldiers would have been killed outright, but there will have been a large group of survivors. The number of military survivors on 1 October 1950 will be estimated separately.

Returning to the civilian populations of Hiroshima and Nagasaki, there were $n_{RS}\left(t_{1950}\right)=195{,}000$ "resident" survivors, where "resident" implies that the survivor was still living in one of the two cities on 1 October 1950, t_{1950}. The LSS Core Cohort will have made up a fraction a_{cc} of this survivor population:

$$n_{cc}\left(t_{1950}\right)=a_{cc}n_{RS}\left(t_{1950}\right). \tag{1}$$

Now imagine, in a thought experiment, that it was possible to identify, immediately after the bombings, all those not destined to die from the blast, from burns and from acute radiation poisoning and that a cohort of these survivors was chosen for health monitoring. Let the new cohort of survivors contain $n_{cpc}\left(t_{1945}\right)$ members at time, t_{1945}, just after the bombings in August 1945. Assume that this new cohort was chosen to be demographically similar to the population of survivors who did not intend moving away from the two cities in the next five years. Alternatively, make the wider assumption (which will be used later in the paper to extrapolate from the LSS Core Precursor Cohort to the other civilian survivor populations) that the new cohort was chosen to be demographically similar to the population of all civilian survivors and that the two populations, of those civilians who would remain resident and of those who would move away, were both demographically similar to the full population of civilian survivors.

Let the number of people in the new cohort make up a fraction a_{cpc} of the total number of survivors in August 1945 $n_{RS}(t_{1945})$ not intending to move away from the cities, so that

$$n_{cpc}\left(t_{1945}\right)=a_{cpc}n_{RS}\left(t_{1945}\right). \tag{2}$$

The number of surviving members of the new cohort will decline over time as a result of normal mortality. Under the assumption of demographic similarity, the relative decline in the cohort will equal the relative decline in the population of resident survivors, so that, as shown in Appendix E, eqn (E.16):

$$\frac{n_{cpc}\left(t\right)}{n_{RS}\left(t\right)}=\frac{n_{cpc}\left(t_{1945}\right)}{n_{RS}\left(t_{1945}\right)} \tag{3}$$

at any time t. Combining eqns (2) and (3) gives

$$n_{cpc}\left(t\right)=a_{cpc}n_{RS}\left(t\right). \tag{4}$$

Hence, putting $t = t_{1950}$, the number of survivors in this new cohort on 1 October 1950 will be

$$n_{cpc}\left(t_{1950}\right)=a_{cpc}n_{RS}\left(t_{1950}\right). \tag{5}$$

Now constrain the selection of the new cohort so that it must contain the same fraction of the resident survivor population in 1945 as does the LSS Core Cohort of the survivor population in 1950, so that:

$$a_{cpc}=a_{cc}. \tag{6}$$

Substituting from eqn (6) into eqn (5) gives:

$$n_{cpc}\left(t_{1950}\right)=a_{cc}n_{RS}\left(t_{1950}\right). \tag{7}$$

Comparing eqns (1) and (7), it is clear that

$$n_{cpc}\left(t_{1950}\right)=n_{cc}\left(t_{1950}\right), \tag{8}$$

so that the number of survivors in the new cohort will have fallen to the number in the LSS Core Cohort by 1 October 1950.

It will be shown in §4.1 that the LSS Core Cohort is made up of most of those in Category 1, most of those in

Category 2, and all of those in Category 3. The age and sex of Category 3 survivors have been matched to those of Category 1 (Ozasa et al., 2018), and so Categories 1 and 3 will have the same demographic structure. Assuming, as seems reasonable, that the population located between 2 and 2.5 km from the hypocentre (Category 2) would have the same demographic structure as those located closer in (Category 1) before the bombings, the indiscriminate nature of the bombs' killing effects would mean the population out to 2 km and that between 2 and 2.5 km would share the same demographic structure after the bombing had occurred, also. The survivors in Category 2 would then have a similar demographic to Category 1 survivors, and this would give the LSS Core Cohort a homogeneous demographic structure.

Even if the Category 2 survivors had a somewhat different make-up, the effect would be relatively small because the LSS Core Cohort is dominated by Categories 1 and 3, who make up four fifths of the total number of members, with Category 2 comprising only a fifth.

It is assumed, in line with Ozasa et al. (2019)—see 1st paragraph of Appendix C—that the LSS Core Cohort will have the same age and sex structure as the cohort of all resident civilian survivors, viz. those survivors still living in one of the two cities on 1 October 1950, who numbered 195,000 at that time. From the arguments above, this demographic similarity will apply, homogeneously, to all Categories within the cohort.

By eqn (E.19) of Appendix E, two populations of survivors that are demographically matched at any time will be matched at all times. Hence the new cohort, which was chosen to be demographically similar, in August 1945, to the population of resident civilian survivors not intending moving away from the two cities, will be still matched demographically to this population as it evolves, and specifically on 1 October 1950, when it will comprise the 195,000 survivors still resident in the two cities. Hence the new cohort will not only contain the same number of survivors as the LSS Core Cohort on 1 October 1950 but it will also be demographically similar: the new cohort will have the same numbers of survivors of both sexes at all ages as the LSS Core Cohort.

There will be a very large number of such new, demographically similar cohorts, each containing the same number of people on 7 August 1945. Some of these new cohorts will contain, as a subset, all the individuals who will later constitute the LSS Core Cohort. Such a cohort, which will contain everyone in the LSS Core Cohort, will qualify as a "LSS Core Precursor Cohort".

Although it would be possible, in theory, to list the members of each LSS Core Precursor Cohort, this would require perfect records to have been kept and it is infeasible in practice. In fact, it will be sufficient, for the purposes of this paper, to calculate the expected number of people in such a cohort on 7 August 1945, as well as their age structure, which will be common to all LSS Core Precursor Cohorts.

The age[10] structure of people in the LSS Core Cohort may be found from the information provided by Ozasa et al. (2012), which specifies the composition of the survivors in the LSS Core Cohort in terms of the ages they would have been at the time the bombs dropped (see Table 1 below). The methods used to calculate the number of survivors and their ages in an LSS Core Precursor Cohort are laid out in Appendices A to D.

Table 1. Age distribution at exposure for the LSS Core Cohort (from Table 2 of Ozasa et al., 2012).

Age/ years at exposure	Number of subjects	Percentage of total
0–9	17,833	20.6%
0–19	17,563	20.3%
20–29	10,891	12.6%
30–39	12,270	14.2%
40–49	13,504	15.6%
>50	14,550	16.8%
Total	86,611	100.0%

It is found that 94,276 people would have been expected to be members of an LSS Core Precursor Cohort on 7 August 1945, the central date of the bombings. 7,665 of these will have been expected to die from normal causes by 1 October 1950, leaving 86,611 people, or just under 92 per cent, still living on that date. The gradual decline in the number of survivors in an LSS Core Precursor Cohort as a result of normal mortality is shown in Fig. 1.

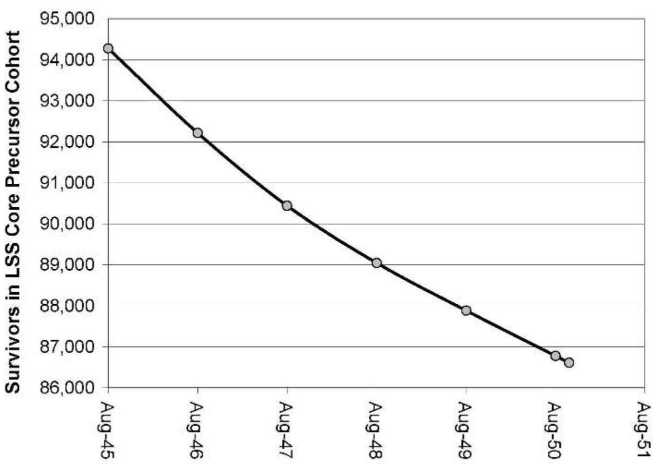

Figure 1. The gradual fall in the number of survivors in an LSS Core Precursor Cohort as it approaches identity with the LSS Core Cohort on 1 October 1950.

[10] In the most general formulation, both age and sex have an effect on survival probabilities, but the approach used here is to use Japan's official life tables for the two sexes combined.

Each LSS Core Precursor Cohort will be a superset of the LSS Core Cohort. For ease of reference, one of these will, notionally, be selected and called "the" LSS Core Precursor Cohort, even though any subsequent deductions will apply equally to all the other LSS Core Precursor Cohorts.

The LSS Core Precursor Cohort, and its subcohorts, assumed demographically similar, may be used as reference cohorts against which other survivor cohorts may be compared.

4. The reduction in radiation dose with distance

Radiation doses close to the hypocentre were extremely high and usually lethal, but survival chances increased with distance. Beebe et al. (1962) suggest that 25 per cent of those irradiated at 1 km recovered, 50 per cent at 1.3 km and 90 per cent at 2 km. Doses at 2.5 km and farther away would have fallen a long way below the level that would cause acute symptoms, so that there would have been no immediate deaths beyond this point.

4.1 The numbers of survivors in the LSS Core Precursor Cohort: those within 2.5 km of the hypocentres ("proximal") and those farther out ("distal")

The LSS Core Cohort was chosen to include all the survivors, still resident in the cities on 1 October 1950, who had been within 2.5 km of the hypocentre, at either Hiroshima or Nagasaki when the bombs dropped, and classed as "proximal". This criterion for selection was adopted to reflect the very strong relationship between distance and radiation dose discussed by Beebe et al. (1962). Almost everyone located beyond 2.5 km ("distal") received only a very low dose. Death in the short term would have been ruled out, as previously noted, and people this distance away faced only a very small risk of succumbing to a radiation-induced cancer at some later time.

The mean weighted dose to the colon, often chosen as a reference, which is made up of the gamma dose plus ten times the neutron dose to allow for neutron radiation's greater biological effect, drops below 5 mGy at locations farther out than 2.5 km from the hypocentre at Hiroshima and at distances of 2.7 km or more from the hypocentre at Nagasaki (Bundesamt fuer Strahlungschutz, 2025).

A similar phenomenon occurs for doses to the bone marrow that are important in the initiation of radiation-induced leukaemia. The rapid fall off in the radiation dose to the bone marrow, with distance, is illustrated in Fig. 2. The graph shows the geometric mean of the upper and lower limits of the marrow dose categories used by Richardson et al. (2009) plotted against the mean distance from the hypocentre, as weighted by person–time. It is clear that the bone marrow dose drops to very low levels at distances of 2 km or more from the hypocentre. The reduction in radiological harm with distance is illustrated in the companion graph, Fig. 3, which plots deaths from radiation-induced leukaemia against the mean distance from hypocentre, again weighted by person–time.

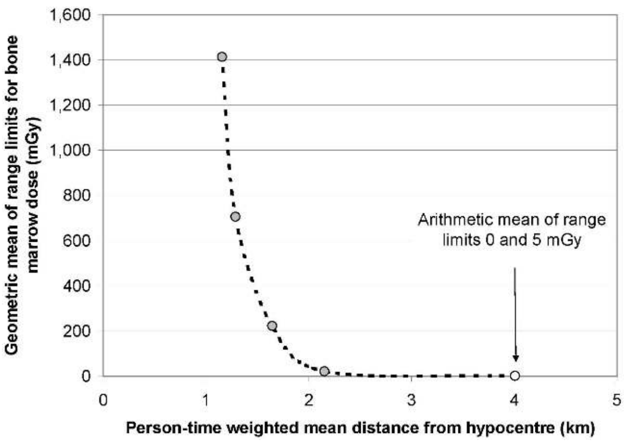

Figure 2. Geometric mean/ arithmetic mean of range limits for bone marrow dose versus person–time weighted mean distance from hypocentre.

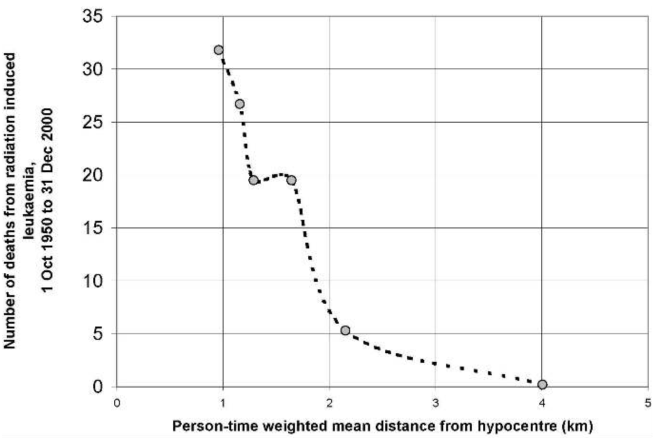

Figure 3. Number of deaths from radiation-induced leukaemia versus person–time weighted mean distance from hypocentre. (The numbers of deaths are as calculated by Richardson et al. (2009), and are based on that paper's dose categories.)

To a good approximation, those located farther than 2.5 km from one of the hypocentres will have received less than 5 mGy of radiation dose to either colon or bone marrow. This may be checked by comparing the numbers of survivors listed in the two groups: (i) "distal" survivors located at distances between 2.5 and 10 km and (ii) survivors receiving less than 5 mGy of colon/marrow dose.

The path to confirmation is complicated by the incomplete specification of the classification by distance, x, from the hypocentre for the members of the LSS Core Cohort. As noted in §1, the numbers given for Category 1 (where $x < 2$ km), Category 2 ($2 \text{ km} < x < 2.5$ km) and Category 3 ($2.5 \text{ km} < x < 10$ km) sum to 93,741, which is greater than the number of survivors, 86,611, assigned to the LSS Core Cohort on 1 October 1950. As noted previously, the difference, 7,130, is accounted for by people within Categories 1, 2 and 3 whose dose records are not good enough for them to be admitted to the LSS Core Cohort. Unfortunately there is a lack of information on how these individuals are distributed amongst those three categories.

However, the survivors in Category 3 were subject to selection, and it may be safely assumed that, when given the choice, the selectors would pick out only those survivors for whom they had adequate dose information. This suggests that the full number, 39,419, of people in LSS Category 3 should be included in the LSS Core Cohort, leaving 86,611 – 39,419 = 47,192 people from Categories 1 and 2 as members of the LSS Core Cohort on 1 October 1950. These "proximal" individuals would have been located within 2.5 km of the hypocentre, and some of them could have experienced very high doses. By this argument, the 7,130 survivors for whom no adequate dose estimate exists would have been proximal, viz. located within 2.5 km of the hypocentre, and they might have received a large dose of radiation when the bombs exploded.

An independent check on this reasoning is available via the table included in Cologne and Preston (2000). This lists 40,403 survivors within the LSS Core Cohort on 1 October 1950 as located between 1.94 and 2.77 km of the hypocentre, and 4899 as positioned between 1.74 and 2.58 km. Making the assumption of a uniform spread of survivors over distance in each of these two categories gives, after adding in the numbers listed for the other distance categories, 45,995 as the number of people in LSS Core Cohort on 1 October 1950, who were located within 2.5 km of the hypocentre. This implies that 40,616 survivors, as of 1 October 1950, were positioned between 2.5 km and 10 km away.

These two figures, 45,995 within 2.5 km and 40,616 between 2.5 km and 10 km, are, in view of the simplifying assumptions made, sufficiently close (within 3%) to the previously derived figures of 47,192 and 39,419 to suggest that the latter figures must be, at least approximately, valid as the numbers of survivors respectively closer than 2.5 km to the hypocentre and farther out.

It may also be noted that the number, 39,419, of people positioned between 2.5 km and 10 km of the hypocentre is also similar to the figure of 38,509 LSS Core Cohort survivors on 1 October 1950 who are listed in Table 9 of Ozasa (2012) as receiving a colon dose of less than 5 mGy. This suggests that $39{,}419 - 38{,}509 = 910$ survivors, as of 1 October 1950, in the LSS Core Cohort who were located more than 2.5 km from the hypocentre received doses higher than 5 mGy to the colon. These people, who represent 2.3 per cent of the number in the LSS Core Cohort located beyond 2.5 km, can be expected to have been positioned between 2.5 km and 2.7 km of the hypocentre at Nagasaki, based on the figures from the Bundesamt fuer Strahlungschutz (2025). Fig. 2 suggests that their average bone marrow dose would have been only slightly higher than 5 mGy.

Appendix F estimates the numbers of radiation-induced cancer deaths, solid and leukaemia, amongst the cohort of survivors, numbering 910 on 1 October 1950, who were positioned beyond 2.5 km but received colon/marrow doses of more than 5 mGy

An estimate was made of the numbers of survivors in the LSS Core Cohort on 1 October 1950 at distance intervals 0–2 km (Category 1) and 2–2.5 km (Category 2) by first dividing the number, 7,130, of people estimated to have been within 2.5 km of the hypocentre, but whose dose is unknown, by making an allocation in proportion to the numbers in Categories 1 and 2. The resultant figures are then subtracted from the numbers in Category 1 and Category 2 respectively. By this model, the number $n_{\text{Cat1}}\big|_{excluded}$ of survivors positioned between 0 and 2 km of the hypocentre but excluded from the LSS Core Cohort for lack of dose information, is:

$$n_{\text{Cat1}}\big|_{excluded} = \frac{34{,}363}{34{,}363 + 19{,}959} \times 7{,}130 = 4{,}510 \quad (9)$$

while the number $n_{\text{Cat2}}\big|_{excluded}$ between 2 and 2.5 km that is excluded from the LSS Core Cohort is

$$n_{\text{Cat2}}\big|_{excluded} = \frac{19{,}959}{34{,}363 + 19{,}959} \times 7{,}130 = 2{,}620. \quad (10)$$

Hence the number in Category 1 (0–2 km) included in the LSS Core Cohort on 1 October 1950 is $34{,}363 - 4{,}510 = 29{,}853$ and the corresponding number in Category 2 (2–2.5 km) is $19{,}959 - 2{,}620 = 17{,}339$.

The lack of follow-up on those who were living away from the cities after 1 October 1950 means that it is not possible to subdivide the estimated 79,278 civilians and 24,269 military personnel who were not resident in the cities on that date into distance categories.

The information contained here and in §3 is summarized in Table 2, which, where possible, splits the survivors into distance categories: those within 2 km of the hypocentres, those between 2 km and 2.5 km, and those who were between 2.5 km and 10 km when the bombs were dropped.

Table 2. Summary of information on distance from hypocentre for all survivors alive on 1 October 1950.

	0–2 km	2–2.5 km	0–2.5 km	2.5–10 km	0–10 km
LSS Core Cohort survivors	29,853	17,339	47,192	39,419	86,611
Civilian survivors resident in cities on 1.10.50 but not in LSS Core Cohort			7,130		7,130
Civilian survivors resident in cities on 1.10.1950, but not in LSS Core Cohort				101,259	101,259
Civilian survivors resident in cities on 1.10.1950 (total)					195,000
Civilian survivors resident outside cities on 1.10.1950					79,278
Military survivors resident outside cities on 1.10.1950					24,269
Total survivors					298,547

5. Leukaemia deaths in the LSS Core Precursor Cohort

5.1 Radiation-induced leukaemia deaths between 1 October 1950 and 31 December 2000

Richardson et al. (2009) estimated that 103 deaths were caused by radiation-induced leukaemia in the LSS Core Cohort between 1 October 1950 and 31 December 2000. Of these, 102.8 were calculated to have occurred in the members of the LSS Core Cohort who were exposed to 5 mGy or more of radiation to the bone marrow, with only a fractional number, 0.2 deaths over the 50 year period, calculated for those who received a dose of less than 5 mGy.

Nearly 95% of the leukaemia deaths occurred at radiation doses of least 100 mGy. Those who received 100 mGy or more were located within average distances of between 0.96 and 1.64 km from the hypocentres, with those closer to the blast receiving doses of over 2000 mGy, well over the threshold for acute symptoms to be experienced. There were only an estimated 5.3 deaths over the 50 year period amongst those who received bone marrow doses of between 5 mGy and 100 mGy, who were located, on average, 2.15 km from the hypocentre.

The next section will quantify how many people died of radiation-induced leukaemia prior to 1 October 1950, a period the Richardson study does not address.

5.2 Radiation-induced leukaemia deaths prior to 1 October 1950 in the LSS Core Precursor Cohort

Ozasa et al. (2012) point to the “unavoidable exclusion” from their study of “perhaps an appreciable number of leukemia cases occurring before 1950”. In fact, sufficient published material exists to make a reasonable estimate of how many radiation-induced leukaemia deaths there would have been at Hiroshima and Nagasaki before 1 October 1950.

Folley et al. (1952) highlight the apparently high incidence of the blood cancer, leukaemia, observing that the number of onsets of leukaemia at Hiroshima and Nagasaki rose above levels normal for Japan from 1948 onwards. Tomonaga (1962) suggests that the increase may have begun in early 1947, but not earlier. Tomonaga gives, in his Fig. 2, the number of leukaemia onsets in the years 1947, 1948, 1949 and 1950 amongst those who were within 2 km of the hypocentres at Hiroshima and Nagasaki. The data are shown in Table 3 below. Taking the number of onsets in the nine months to 1 October 1950 to be $\frac{3}{4} \times 19 = 14.25$ gives a total of 46.25 inceptions during the 3¾ year period between 1 January 1947 and 1 October 1950.

Table 3. Onsets of leukaemia at Hiroshima and Nagasaki (from Fig. 2 of Tomonaga et al., 1962).

	1947	1948	1949	1950	Total
Hiroshima	2	11	8	11	32
Nagasaki	3	5	3	8	19
Total	5	16	11	19	51

Watanabe et al. (1960) report that, during the period 1946–1951 at Hiroshima, there were 50 onsets of leukaemia within 2 km of the hypocentre at Hiroshima, but only 34 deaths. Applying the resultant ratio, 34/50 = 0.68, of deaths to onsets to the two cities for the included period 1 January 1947 to 30 September 1950 suggests that there would have been $0.68 \times 46.25 = 31.45$ leukaemia deaths during this time. This number comprises both naturally occurring and radiation-induced leukaemia deaths across the population of survivors who were located within 2 km of the hypocentres in the two cities.

Resident survivors within 2 km of the hypocentres would all be members of Category 1, as defined in §1, of which there were 34,363 members on 1 October 1950. However, a lack of dose data means that the number of survivors within 2 km of the hypocentre in the LSS Core Cohort is smaller, at that time, at 29,853 (as discussed in §4.1 above and listed in Table 2). The number of leukaemia deaths, natural and radiation-induced, amongst the LSS Core Cohort of survivors between 0 and 2 km of the hypocentre may thus be estimated as:

$$\frac{29{,}853}{34{,}363}\times 31.45 = 27.3. \qquad (11)$$

Richardson et al. (2009) provide, in their Table 4, a breakdown of deaths from naturally occurring leukaemia ("fitted background") for the five ten-year intervals between 1950 and 2000. Their Table 1 gives the number of person–year in each decade for the LSS Core Cohort, which allows the naturally occurring rate per person per year to be calculated as a function of time, as shown in Fig. 4. The rate plateaus in the1950s, which suggests that the incidence of naturally occurring leukaemia deaths for 1950 to 1960, 4.75×10^{-5} per person per year, may be applied to the previous 3.75 year period, 1 January 1947 to 30 September 1950.

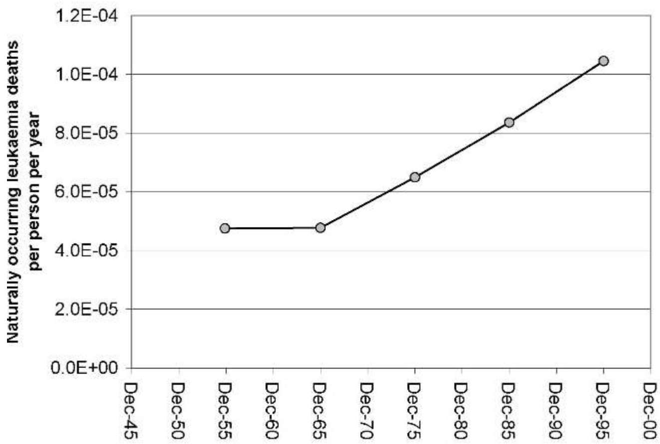

Figure 4. Death rate from naturally occurring leukaemia per person per year, based on Tables 1 and 4 of Richardson et al. (2009).

The number of LSS Core Cohort Category 1 survivors, who were located within 2 km of the blasts, may be calculated at dates earlier than 1 October 1950 under the assumption of demographic homogeneity among subcohorts. The number of Category 1 survivors on earlier dates will be proportional to the number of survivors in the LSS Core Precursor Cohort as a whole, which varies over time as shown in Fig. 1. Thus, for example, there would have been 31,260 Category 1 survivors in the LSS Core Precursor Cohort at the beginning of 1947, a number which will have dwindled to 30,335 two years later, before falling to 29,853 on 1 October 1950.

The number of person years for the 3.75 year period from the date, 1 January 1947, cited by Tomonaga as the earliest at which a radiation-induced leukaemia death could have occurred, up to 1 October 1950, may then be estimated as 1.15×10^{5}. Multiplying this figure by the incidence, 4.75×10^{-5} per person per year, gives 5.5 naturally occurring leukaemia deaths amongst the LSS Category 1 survivors. Subtracting this number from the 27.3 deaths from leukaemia in total, derived in eqn (11), gives ~22 deaths from radiation-induced leukaemia in the period before 1 October 1950 amongst survivors out to 2 km in the LSS Core Cohort.

The number of radiation-induced leukaemia deaths becomes very small at distances beyond 2 km because of the sharp decline in dose with distance, as illustrated by Fig. 2. Richardson et al. (2009) estimated that only about 5 per cent of radiation-induced leukaemia deaths occurred beyond 2 km. Adding on 5 per cent leads to an estimate of the number of radiation-induced leukaemia deaths in the LSS Core Cohort between 1 January 1947 and 1 October 1950 to rise by one to 23. Accepting Tomonaga's conclusion that no radiation-induced leukaemia deaths occurred prior to 1 January 1947, the number of radiation-induced leukaemia deaths between August 1945 and 1 October 1950 takes the same value:

$$n_{radleuk}\big|_{1945-1950} = 23. \qquad (12)$$

5.3 Radiation-induced leukaemia deaths over all time in the LSS Core Precursor Cohort

The probability distribution methods described in Appendix G may be used to estimate the number of leukaemia deaths there will have been to 1 January 2056, beyond which time all survivors would be expected to have died whether or not they were exposed to radiation. Japanese life tables do not extend past 109, and the date, just over 110 years after the bombs dropped, was chosen in acknowledgement of this approximate limit to longevity.

The calculations have been carried out under the assumptions that

- there were 23 radiation-induced leukaemia deaths between 7 August 1945 and 30 September 1950
- that the numbers of radiation-induced leukaemia deaths in the five decades between 1 October 1950 and 31 December 2000 are as found by Richardson et al. (2009).

Two candidate probability distributions were used to characterize the random latency time between receiving the radiation dose and death, should it occur, from radiation-induced leukaemia: the Weibull and the lognormal. The lognormal was found to represent the data better, and Fig. 5 shows the optimal match, produced by adjusting the median latency time, ϕ, and the standard ratio, ρ, to minimize the χ^2 statistic (eqn G.11).

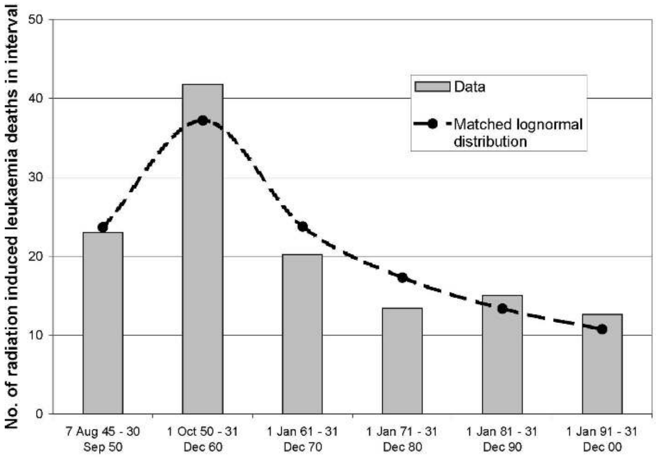

Figure 5. Matching the lognormal distribution to the data for radiation-induced leukaemia deaths.

The optimal values found for the median ϕ and the standard ratio ρ of the latency interval T were:

$$\begin{aligned} \phi &= e^m = 47.14 \text{ years} \\ \rho &= e^s = 5.67 \text{ (dimensionless)} \end{aligned} \tag{13}$$

where m is the mean (and median) of $\ln(T)$, while s is the standard deviation of $\ln(T)$. The standard ratio is analogous to the standard deviation in the normal distribution, except that its rôle is multiplicative rather than additive. Thus the probability of T lying in the interval $\rho^{-1}\phi < T \le \rho\phi$ is 0.6826, the probability that $\rho^{-2}\phi < T \le \rho^2\phi$ is 0.9544 and so on, figures that will be familiar from normal distribution tables.

Fig. 6 shows the predicted deaths per year from radiation-induced leukaemia out to August 2055.

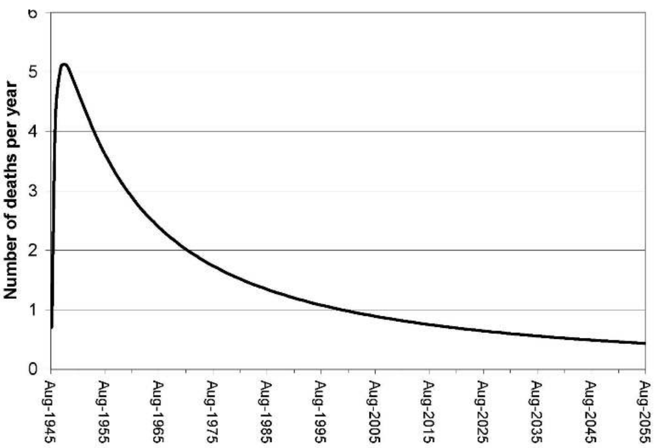

Figure 6. Central estimate of the number of deaths per year from radiation-induced leukaemia.

Using the concept of q-values, which are introduced in Appendix G as analogues of p-values when the given data are calculated values rather than raw observations, gives the results summarized in Fig. 7. The graph shows the locus of $q = 0.05$ on the plane of the standard ratio, ρ, and the median, ϕ, with the optimal point, which has a q-value of 0.47, also plotted.

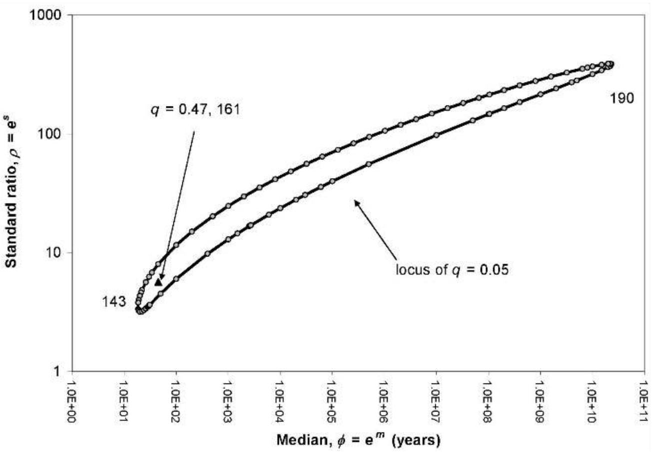

Figure 7. Optimal q-value and contour of $q = 0.05$ in the plane of the standard ratio and the median; the number of deaths from radiation-induced leukaemia, most likely and bounding values, are also shown.

While the lowest value of ϕ consistent with $q = 0.05$, is 18.5 years, the highest value is over 10^8 years. The latter implies the distribution would have a very long tail, see Fig. 8. However, the pragmatic assumption that no one will live beyond the age of 110 means that the tail is truncated in 2055, and this restricts the associated number of radiation-induced leukaemia deaths to under 200.

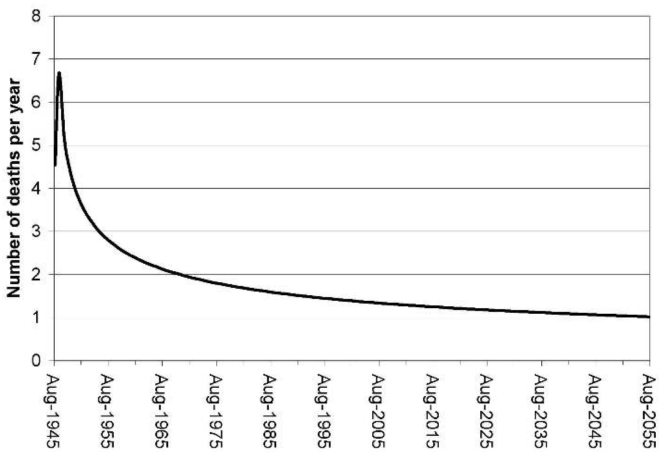

Figure 8. Annual number of radiation-induced leukaemia deaths: upper bound.

The optimal value of the number of deaths from radiation-induced leukaemia to 1 January 2056 is

$$n_{radleuk}\big|_{1945-2056} = 161. \tag{14}$$

This figure is subject to a lower bound of 143 and an upper bound of 190.

5.3.1 Using only the data from 1 October 1950 to 31 December 2000

This subsection's approach takes no account of the estimate of pre-1950 deaths, and uses only the data from Richardson et al. (2009) on the numbers of deaths by decade to find the parameters. Those to be found have increased in number from two to three, and they are the median latency interval and the standard ratio of the latency interval as before but now, also, the number of deaths from radiation-induced leukaemia in the interval 7 August 1945 to 1 October 1950. The resultant values are:

$$\begin{aligned} &\phi = 29.65 \text{ years} \\ &\rho = 4.60 \text{ (dimensionless)} \\ &n_{radleuk}\big|_{1945-1950} = 24.6 \end{aligned} \qquad (15)$$

The value found for $n_{radleuk}\big|_{1945-1950}$ is clearly close to the value of 23 found in §5.2 for radiation-induced leukaemia deaths before 1 October 1950. The corresponding number of radiation-induced leukaemia deaths in the LSS Core Precursor Cohort is then:

$$n_{radleuk}\big|_{1945-2056} = 156 \qquad (16)$$

which is again similar to, and so provides confirmation for, the value, 161, given in eqn (14).

5.3.2 Radiation-induced leukaemia deaths between 1 January 2001 and 31 December 2003

It is useful to extend the period covered by the Richardson et al. (2009) study of leukaemias by two years to allow for a direct comparison with the figures for solid cancers contained in Ozasa et al. (2012), which estimates the number of radiation-induced solid cancer deaths in the LSS Core Cohort there will have been between 1 October 1950 and 31 December 2003.

The best-estimate model for radiation-induced leukaemia, with the distribution parameters defined by eqns (13), suggests that

$$n_{radleuk}\big|_{2001-2003} = 2.8 \qquad (17)$$

so that the total number of deaths from radiation-induced leukaemia from 1 October 1950 to 31 December 2003 is

$$n_{radleuk}\big|_{1950-2003} = 105.8. \qquad (18)$$

6. Radiation-induced leukaemia deaths across all cohorts and time periods

Appendix I details the calculations of deaths, by time period, from radiation-induced leukaemia for:

- the civilian survivors resident in one of the two cities on 1 October 1950 but not in the LSS Core Cohort, proximal (< 2.5 km from the hypocentre) and distal (2.5–10 km away)
- the civilian survivors who were living away from the cities by 1 October 1950

and

- the military survivors who were living away by 1 October 1950.

The resulting numbers of deaths by time period for these groups, as well as for the LSS Core Precursor Cohort, are given in Table 4.

Table 4. Summary by time period of estimated deaths from radiation-induced leukaemia amongst the survivors of the Hiroshima and Nagasaki bombings.

	Survivors in August 1945	Survivors on 1 Oct 1950	Deaths 1945–1950	Deaths 1950–2000	Deaths 2001–2003	Deaths 1945–2003	Deaths 1945–2055	Total deaths as % of survivors Aug 1945
LSS Core Precursor Cohort	94,276	86,611	23	103	2.8	128.8	161	0.17%
Proximal resident civilians*	7,761	7,130	3.5	15.5	0.4	19.4	24.2	0.31%
Resident distal civilians*	110,220	101,259	0.2	0.9	0.0	1.2	1.4	0.001%
Resident civilians subtotal	212,257	195,000	27	119	3	149	187	0.09%
Civilian survivors who were living away	86,294	79,278	10.8	48.6	1.3	60.7	75.9	0.09%
Military survivors who were living away	25,450	24,269	3.3	14.9	0.4	18.6	23.2	0.09%
Subtotal of those living away	111,744	103,547	14	63	2	79	99	0.09%
Overall total	324,000	298,547	41	183	5	229	286	0.09%

* Not included in LSS Core Precursor Cohort.

7. Radiation-induced solid cancer deaths in the LSS core and core precursor cohorts

7.1 Radiation-induced solid cancer deaths between 1 October 1950 and 31 December 2003 in the LSS Core Cohort

Ozasa et al. (2012) estimate that 527 people died from radiation-induced solid cancer in the period 1 October 1950 to 31 December 2003.

They put the number into the context of the almost twenty times greater number, 10,402, of deaths from solid cancers in the same period that were unrelated to radiation exposure. Ozasa et al. note also that the total number of deaths recorded amongst the LSS Core Cohort between 1 October 1950 and 31 December 2003 was 46,614.

7.2 Radiation-induced solid cancer deaths over all time in the LSS Core Precursor Cohort

Applying the probability methods described in Appendix G to data given in Furukawa et al. (2009) and Double et al. (2012) gives a central prediction of 1,585 deaths from radiation-induced solid cancer to the end of 2055 for the LSS Core Precursor Cohort. Appendix H gives fuller details of the calculation.

8. Radiation-induced solid cancer deaths across all cohorts and time periods

Appendix J explains the calculations by time period for:

- the civilian survivors resident in one of the two cities on 1 October 1950 but not in the LSS Core Cohort, proximal and distal
- the civilian survivors who were living away by 1 October 1950, and
- the military survivors who were living away by 1 October 1950.

The resulting numbers of deaths from radiation-induced solid cancer for these groups, as well as for the LSS Core Precursor Cohort are given, by time period, in Table 5.

9. Total deaths at the two cities from radiation-induced cancer, both leukaemia and solid cancer

Table 6 gives the estimated number of deaths from both forms of cancer, broken down by cohort and time period.

Table 5. Summary by time period of estimated deaths from radiation-induced solid cancers.

	Survivors in Aug 1945	Survivors on 1 Oct 1950	Deaths 1945– 1950	Deaths 1950–2003	Deaths 1945–2003	Deaths 1945–2055	Total deaths as % of survivors Aug 1945
LSS Core Precursor Cohort	94,276	86,611	0	527	527.0	1585	1.68%
Proximal resident civilians*	7,761	7,130	0	79.1	79.1	237.9	3.07%
Resident distal civilians*	110,220	101,259	0.0	9.0	9.0	26.9	0.02%
Resident civilians subtotal	212,257	195,000	0	615.1	615.1	1,850	0.87%
Civilian survivors who were living away	86,294	79,278	0	250.1	250.1	752.1	0.87%
Military survivors who were living away	25,450	24,269	0	76.5	76.5	230.2	0.90%
Subtotal of those living away	111,744	103,547	0	327	327	982	0.88%
Overall total	324,000	298,547	0	942	942	2,832	0.87%

* Not included in LSS Core Precursor Cohort.

Table 6. Summary by time period of estimated deaths from all radiation-induced cancers, leukaemia and solid cancers.

	Survivors in August 1945	Survivors on 1 Oct 1950	Deaths 1945–1950	Deaths 1950–2003	Deaths 1945–2003	Deaths 1945–2055	Total deaths as % of survivors Aug 1945
LSS Core Precursor Cohort	94,276	86,611	23	633	656	1,746	1.85%
Proximal resident civilians	7,761	7,130	3	95	98	262	3.38%
Resident distal civilians	110,220	101,259	0	10	10	28	0.03%
Resident civilians subtotal	212,257	195,000	27	738	764	2,037	0.96%
Civilian survivors who were living away	86,294	79,278	11	300	311	828	0.96%
Military survivors who were living away	25,450	24,269	3	92	95	253	1.00%
Subtotal of those living away	111,744	103,547	14	392	406	1,081	0.97%
Overall total	324,000	298,547	41	1,129	1,170	3,118	0.96%

10. Noncancer mortality

35,685 of the 86,611 members of the LSS Core Cohort had died from causes unrelated to cancer or leukaemia between 1 October 1950 and 31 December 2003. Of these thirty five thousand, Ozasa et al. (2012) estimated that 353 might have died of noncancer deaths associated with exposure to radiation. The excess relative risk (ERR) was found to be zero (or slightly negative) between 1950 and 1965 for doses up to 1500 mGy, a very high dose that would have been experienced only by those positioned within about 1 km of the hypocentre and that would have caused acute radiation syndrome at the time. However, the ERR was calculated to rise roughly linearly with dose between 1966 and 2003, which might suggest a delay of twenty years before the long-term, noncancer effects of radiation were felt (Fig. 6A of Ozasa et al., 2012), rather in the way that no radiation-induced solid cancer deaths were found until at least 10 years after exposure (Marshall et al., 1983).

The noncancer deaths associated with radiation exposure represent a sizeable fraction, 56%, of the radiation-induced cancer deaths in the LSS Core Cohort over the interval 1 October 1950 to 31 December 2003, which totalled 633 when leukaemia deaths are added to those from solid cancers (see Table 6).

But despite the apparently firm estimate of 353 radiation-induced noncancer deaths in Ozasa et al. (2012), seven years later Ozasa et al. (2019) were suggesting only that “Epidemiological studies have observed ... possible risks for some noncancer diseases” and saying that “Radiation effects on noncancer outcomes are a current focus” for the Radiation Effects Research Foundation. The reservation is a reflexion of the fact that, while in-cell mechanisms have been advanced to explain how radiation can cause cancers, no causal mechanism has yet been identified to link noncancer deaths with radiation.

11. Life expectancy of survivors at Hiroshima and Nagasaki

Cologne and Preston (2000) studied how the long-term risks to health after the bombings might affect longevity, noting that: “Compared with the dramatic early effects that resulted in death soon after exposure (such as acute radiation syndrome or leukaemia), a large excess relative or absolute rate for effects occurring late in life does not imply a large decrease in individual length of life.” A similar point is made in Thomas (2017), where the Marshall model for radiation risk is used to show that the average age at death for those people who died from a radiation-induced cancer was between 60 and 80, irrespective of the duration of the dose or its size, given that it did not cause death from acute radiation syndrome.

Cologne and Preston chose, as their reference group for life expectancy, people from the LSS Full Group (which numbered 120,321 on 1 October 1950), who were in one of the two cities at the time of the bombings, but who received radiation doses of less than 5 mGy, and so would have suffered negligible shortening of their lives as a result of nuclear radiation. The median expected age at death for these individuals, over 90 per cent of whom were positioned between 2.8 km and 10 km from the hypocentre when the bombs fell, was found to be 81 years 30 days.

They found that the individuals in the LSS Full Group who had received at least 1000 mGy of radiation (which means that they would have suffered, but recovered, from acute radiation sickness) and, on average, 2250 mGy, could be expected to die at a median age of 78 years 169 days. This implied a loss of life expectancy of 2.6 years compared with their co-residents who had been exposed to less than 5 mGy. The expected median age at death for the LSS Full Group was found to be 80 years 265 days, which represented a loss of about 4 months compared with the reference group.

The authors note, however, that “Because the LSS cohort was intentionally constructed to include a greater proportion of high-dose survivors, the average loss of life among the larger population of all atomic-bomb exposed individuals who survived acute causes of death would be less than 4 months.” The figures for expected age at death provided by Cologne and Preston (2000) have been used to produce Table 7, which suggests that the loss of life expectancy for all the bomb survivors would, in fact, have been about 1.5 months.

The calculation of loss of life expectancy caused by the bombs is affected strongly by the choice of the reference population. Suppose, for example, that the reference population had been chosen to be only those twenty five and a half thousand people who were members of the LSS Full Group who were positioned between 3 km and 10 km from the hypocentre. There would seem to be good grounds for selecting this new set of people, who made up 75% of the group eventually chosen by Cologne and Preston, because they lived in one of the two cities in question and they received a radiation dose that was certainly less than 5 mGy. But, for whatever reason, their expected median lifetime was found to be 80 years and 322 days, which was less than the expected median lifetime of those who received doses between 5 mGy and 250 mGy, which would appear to suggest that radiation at these levels brought net benefits. Cologne and Preston counsel against this interpretation and against this choice of reference group,

Table 7. Expected age at death for survivors alive on 1 October 1950, based on figures given in the table in Cologne and Preston (2000).

	Survivors on 1 October 1950	Expected age at death		Loss of life expectancy (days)
		years	days	
Distal residents >2.8 km from hypocentre (Reference)	30,945	81	30	0
LSS Core Precursor Cohort proximal	47,192	80	250	146
LSS Core Precursor Cohort distal	39,419	81	21	9
LSS Core Precursor Cohort total	86,611	80	312	83
Resident proximal civilians*	7,130	80	345	50
Resident distal civilians*	101,259	81	21	9
Resident civilians total	195,000	80	352	43
Civilians who were living away	79,278	80	352	43
Military who were living away	24,269	80	352	43
Those living away subtotal	103,547	80	352	43
Overall total	298,547	80	352	43

* Not included in LSS Core Precursor Cohort.

but the effect serves to illustrate just how small were the differences in expected lifetimes between those who had received smallish doses and those who were exposed to only a negligible dose of radiation.

The absolute values of the survivors' expected ages at death are themselves highly significant, however, even after account is taken of the wide error bounds on the expected median age at death, which Cologne and Preston suggest are typically ±5 years. Many bomb survivors, even those who received very large doses, were found to be living well into their seventies or longer, which is decades more than the average life expectancy at birth in Japan at the time the bombs dropped, which was less than 50 years (Ministry of Health, Labour and Welfare, 2025).

12. Suicide risk amongst survivors

Amano et al. (2021) "sought to determine whether measures of exposure severity, as indirect measures of psychological trauma arising from exposure to the atomic bombings, are associated with suicide mortality among atomic bomb survivors".

They concluded that "Proximity to the hypocentre, shielding and acute injury presence do not generally appear to influence suicide mortality among atomic bomb survivors."

13. Children of survivors at Hiroshima and Nagasaki

Ozasa et al. (2018) report, that "Genetic studies have indicated no significant associations between parental exposure to atomic bomb radiation and the frequency of genetic abnormalities among children conceived after the exposure." They also say: "Epidemiological follow-up has indicated no increased risk of cancer or non-cancer disease mortality or cancer incidence associated with parental radiation dose."

14. Discussion

While the RERF studies take, as their point of departure, Japan's census date, 1 October 1950, this paper has introduced the concept of the LSS Core Precursor Cohort to explore the time interval between the dropping of the bombs in early August 1945 and 1 October 1950. Actuarial methods, involving life tables, have been used to establish that the LSS Core Precursor Cohort, a superset that contains all the members of the LSS Core Cohort, had 94,276 members immediately after the bombings. 7,665 of these people will have died by 1 October 1950, overwhelmingly from causes unrelated to radiation, so that the LSS Core Precursor Cohort then degenerates to the LSS Core Cohort, which contains 86,611 survivors at that point.

The study uses, first, the LSS Core Precursor Cohort and, later on, probability distributions, to extend the RERF results for the long-term effect of radiation on the LSS Core Cohort to cover the whole period from immediately after the bombs dropped in August 1945 to the end of 2055, when any remaining survivors would be at least 110 years old. It is found that there will have been 161 deaths from radiation-induced leukaemia in the LSS Core Precursor Cohort and 1585 from radiation-induced solid cancer by 31 December 2055, making a total of 1,746 radiation-induced cancer deaths in the LSS Core Precursor Cohort.

The LSS Core Cohort comprises a sample that was drawn up, retrospectively, from only the population of survivors who were still resident in one of the two cities on 1 October 1950. Two large "resident" populations were omitted from the LSS Core Cohort:

(i) 7,130 "proximal" resident survivors, as of 1 October 1950, who were within 2.5 km of the hypocentre, but for whom there was inadequate positional/dose data, and

(ii) 101,259 "distal" resident survivors, as of 1 October 1950, who were positioned between 2.5 km and 10 km from the blasts.

The numbers of radiation-induced cancer deaths within these "omitted" populations were calculated using the proximal and distal subcohorts respectively of the LSS Core Precursor Cohort as reference populations.

103,547 survivors (79,278 civilian and 24,269 military) are calculated to be living away from the two cities by 1 October 1950. The likely numbers of deaths from radiation-induced solid cancer and leukaemia in these two "living away" populations were estimated by reference to the numbers of deaths calculated for the whole of the civilian population still resident in one of the two cities on 1 October 1950. The strategy will give good accuracy for the civilians who were living away, but demographic differences mean that the results for the military survivors, who dispersed from Hiroshima to the rest of Japan once the war was over, will be more approximate. Any adverse effect on the accuracy of the overall figures for radiation-induced cancer deaths will, however, be limited because the military survivors make up less than ten per cent of the total survivor population.

Predictions of future deaths from radiation-induced leukaemia were made using probability distributions. These distributions were matched to the calculated numbers of deaths over five decades up to 31 December 2000 (Richardson et al., 2009), which were treated as quasi-observations. A new development, the q-value, was then applied to quantify the goodness of fit of the predictions to existing data in a way that took account of both the number of available data points and the number of parameters that needed to be estimated. The χ^2 statistic was calculated and, just as the smallest χ^2 will result in the largest p-value for observed data, so the lowest χ^2 leads to the highest q-value when the data consist of calculated values. The $q = 0.05$ contour then allows reasonable upper and lower bounds to be estimated. The resultant q-values need to be regarded as useful indications, however, rather than strict probabilities.

The number of deaths from radiation-induced leukaemia over all time in the LSS Core Precursor Cohort was found to lie in the interval 143 to 190, with 161 as the most likely value, based on the use of a lognormal distribution and the application of q-value analysis. Truncation at the date 2055, where the youngest possible survivor will be more than 110 years of age, restricts the upper bound to be only 12 per cent higher than the central value. The total number of deaths from radiation-induced leukaemia amongst all the 324,000 survivors, as of August 1945, is then calculated to be 286, implying a roughly one in a thousand chance of dying from this cause (Table 4).

A similar method was used to estimate the number of deaths from radiation-induced solid cancer, using calculated estimates of annual deaths up to 2026 from Furukawa et al. (2009). This time a Weibull probability distribution gave the closest fit to the data. It was found that there will have been between 1287 and 2132 deaths from this cause before 2056, with a central estimate of 1585. Once again the truncation at the end of 2055 restricted the upper bound, which is about a third greater than the central estimate. The total number of deaths from radiation-induced solid cancer in the full cohort of 324,000 long-term survivors, as of August 1945, was found to be 2,832, or 0.87 per cent (Table 5).

Adding together the contributions gives the total number of deaths from radiation-induced cancer, leukaemia or solid, amongst the 324,000 survivors as 3,118, or just under one per cent.

Evidence from epidemiological studies points to increased risks in the long term from noncancer diseases associated with the radiation released by the bombs. Mortality from radiation-induced noncancer disease has been calculated to be about half that of radiation-induced solid cancer and leukaemia, combined. However, while in-cell mechanisms have been advanced to explain how radiation can cause cancer, no similar causal mechanism has yet been identified to link noncancer deaths to radiation.

A potentially confounding factor is the strong dependence of radiation dose on distance from the hypocentre, so that, for example, those nearer the blast might have been exposed to higher physical and mental stresses as well as to higher radiation doses. Lack of data might preclude the option, but it would be good to see a comparison with the long-term effects on the mortality of those who survived the near-contemporaneous Tokyo fire bombing of 9–10 March 1945, in which it is estimated that 80,000 to 100,000 people died.[11] It is possible that dust and smoke, for example, could be an alternative explanation for the small increase in noncancer mortality observed in the LSS Core Cohort.

An important perspective on the long term effect of radiation on survivors' health is offered by the study by

[11] Britannica, Bombing of Tokyo, https://www.britannica.com/event/Bombing-of-Tokyo (accessed August 2024).

Cologne and Preston (2000). Using Cologne and Preston's figures, the overall expected median lifetime for all 298,547 survivors still living on 1 October 1950 was calculated to be 80 years and 352 days, which is only 43 days less than the median lifetime of the reference group selected by Cologne and Preston, which was the members of the LSS Full Group who received a radiation dose of 5 mGy or less. See Table 7. It is also striking that the overall figure for the survivors was only about 2½ months less than Japan's life expectancy at birth in 2000, which was 81 years 66 days in the year when Cologne's and Preston's paper was published (National Institute of Population and Social Security Research, Japan, 2023).

Even those who had absorbed thousands of mGy, would, once they had recovered from the acute radiation sickness inevitable after such very large doses, go on to live into their late seventies. Cologne and Preston found that the individuals in the LSS full cohort who had received at least 1000 mGy of radiation and, on average, 2250 mGy could be expected to die at a median age of 78 years 169 days. This figure for atom bomb survivors with a very high radiation dose is 263 days more than the UK's life expectancy at birth in 2000, which was 77.74 years, and nearly two years more than the corresponding figure for the USA in 2000, which was 76.64 years (World Bank, 2025).[12]

Given that life expectancy was under 50 years in Japan at the time the bombs dropped (Ministry of Health, Labour and Welfare, 2025), the statistical evidence makes it clear that the population of survivors, including those who had received extremely high doses of radiation, and suffered acute radiation syndrome as a consequence, shared in the dramatic increase in longevity that accompanied Japan's post-war economic boom, to which they will, of course, have contributed.[13]

Those who survived the immediate destruction the atomic bombs wrought could expect to live decades longer than their forebears, as demonstrated in Fig. 9, which is based on Table 2 of Ozasa et al. (2012). The columnar graph shows the percentage of members of the LSS Core Cohort in the age ranges 0–9, 10–19, 20–29, 30–39 and 40–49 at the time of the bombings who had survived to at least the age shown on the horizontal axis by 1 January 2004. Thus, for example, over 50 per cent of those who were aged between 20 and 29 at the time of the bombings attained an age of 78.4 years or more, which exceeds the life expectancy at birth for Japanese males born in 1999. One per cent of those aged 40 to 49 at the time of exposure, that is to say 135 people in this age group, were over 98 years old on 1 January 2004.

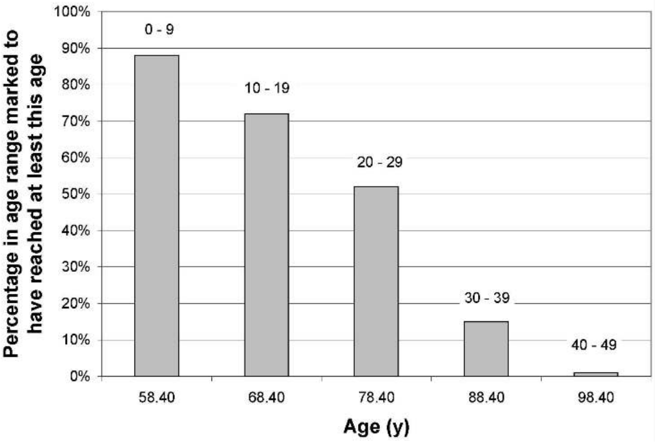

Figure 9. The percentage of those in the age ranges 0–9, 10–19, 20–29, 30–39 and 40–49 at the time of the bombings to have survived to at least the age shown by 1 January 2004. Based on Table 2 of Ozasa et al. (2012).

The analysis of life expectancy shows that, while the bomb survivors experienced a real increase in risk as a result of their radiation dose, the effect was hugely outweighed by the benefits of living in and contributing to a society whose prosperity was growing very rapidly. An intuitive realization of this fact might lie behind the inability by Amano et al. (2021) to find any evidence for an increase in suicides after the bombing among the survivors.

No evidence has been found for an increase in genetic abnormalities in the children of the survivors, nor in the risk of disease for the children of atomic bomb survivors who were conceived after the exposure (Ozasa et al., 2018).

International studies, which have continued over a period of more than 70 years, have subjected the health of a large group of atomic bomb survivors to intense scrutiny since the 1950s. The main finding is that exposure to radiation causes the risk of leukaemia and solid cancer to increase, but that the extra risk is small. This work has shown that a bomb survivor from one of the two cities will have experienced, on average, a roughly 1 per cent increase in the chance of dying from leukaemia or solid cancer. The probability of a typical survivor dying from leukaemia or solid cancer will increase to about 1.3 per cent if both upper bounds from q-value analysis are invoked, and decrease to about 0.8 per cent if the lower bounds are applied.

Investigations are continuing into whether the burst of radiation experienced by the survivors at the time the bombs exploded has caused an increase in noncancer mortality. Despite the lack of an identified causal

[12] World Bank, 2025, Life expectancy at birth, total (years), https://data.worldbank.org/indicator/SP.DYN.LE00.IN?locations= (accessed March 2025).

[13] The very large growth in GDP per head that Japan experienced from the late 1940s onwards will likely have been the dominant factor behind Japan's increase in life expectancy (Thomas and Waddington, 2017). See also Thomas (2017a).

mechanism, epidemiological research suggests there may be a link, subject to a delay of about 20 years. The putative, noncancer mechanism would add about 0.5% to the chance of a survivor dying as a result of radiation exposure. Deaths from this cause would be delayed, so that the effect of late, noncancer deaths on life expectancy would be small. Noncancer deaths caused by radiation were, in fact, allowed for in the life expectancy study by Cologne and Preston (2000), and this allowance is continued in §11 above and in Table 7.

There has, in the past, been a great deal of public concern, not only over the unquestionably enormous destructive power of the atomic weapons used at Hiroshima and Nagasaki, but also over what might be the long-term health effects of the radiation the bombs released on those who survived the blasts. Frank Barnaby, who was director of the Stockholm International Peace Research Institute at the time, expressed the thoughts of many when he wrote in the *New Scientist* in 1977: "The really unique, and perhaps most terrifying, consequences of the atomic bombs are the delayed effects." His opinion hardened over the years, so that, eighteen years later, he wrote: "The delayed effects are the most terrifying consequences of the use of nuclear weapons." (Barnaby 1977, 1995). But the extensive research carried out in the intervening years means that we are now, 80 years on from the bombings, in a better position to quantify more fully how harmful these delayed effects have been at Hiroshima and Nagasaki.

It has become clear that the survivors faced a chance of dying from radiation-induced cancer, solid cancer or leukaemia, in the range 0.8 to 1.3 per cent. To put these figures into a Japanese context, cancers accounted for 30.3 per cent of all deaths across both sexes in Japan in 1996 (Tomonaga, 1999).

Adding on half a per cent to account for the delayed noncancer deaths, identified by epidemiological studies, suggests the chance of dying prematurely will lie somewhere between 1 per cent and 2 per cent. This puts the long term risk well below that of man-made air pollution in an industrialized society, where Public Health England (PHE) states "it is estimated that long-term exposure to man-made air pollution in the UK has an annual effect equivalent to: 28,000 to 36,000 deaths".[14] Given that there were 616,014 deaths across the United Kingdom in 2018,[15] the year PHE first published its information online, this implies that man-made air pollution was causing between 4.6 and 5.8% of all British deaths at that time.

The figures for deaths from radiation amongst the survivors are, of course, all estimates, both those in the RERF studies and those additional figures derived in this paper. The RERF figures are, however, based on careful, statistical analysis of real epidemiological data, and this paper has used the RERF results as the basis for its own analysis. Actuarial methods based on Japanese life tables have been used in this study to extend the analysis back to the date of the bombing via the concept of a precursor cohort. The extension forward to 2055 has made use of concepts used widely in reliability engineering (e.g. Kumamoto and Henley, 1996). While complete accuracy cannot be claimed, nevertheless the techniques used give a strong indication of the size of the risk faced by the survivors of the bombings, including those who received very large doses but then recovered from the acute radiation sickness those doses caused.

15. Conclusions

The paper has expanded the RERF studies from their previous starting date, 1 October 1950, back to August 1945, when the atomic bombs were dropped, and forward to 2055, when the youngest possible survivor will be 110 years of age, which may be regarded as an approximate limit to longevity in Japan. It has also extended the results from the LSS Core Cohort, which contained 86,611 members on 1 October 1950, to the full population of survivors, calculated to number 298,547 on 1 October 1950.

The concept of an LSS Core Precursor Cohort has been introduced, and actuarial methods have been used to find the number of survivors there would have been in a predecessor to the LSS Core Cohort, if the selection criteria for that group had been applied immediately after the bombings, in August 1945, rather than on 1 October 1950. The LSS Core Precursor Cohort emerges as a superset of, and hence a generalization of, the LSS Core Cohort. The concept of a precursor cohort is extended to the other survivor cohorts under the assumption of demographic similarity, which is likely to be at least approximately true.

The number of radiation-induced leukaemia deaths across the two cities in the period before 1 October 1950 were particularized to the LSS Core Precursor Cohort, so that the numbers of deaths from radiation-induced leukaemia, in the LSS Core Precursor Cohort, became available over an unbroken series of time intervals, from the time the bombs dropped to the end of 2000.

[14] Public Health England (PHE), 2018, Health matters: air pollution, https://www.gov.uk/government/publications/health-matters-air-pollution/health-matters-air-pollution . Published 14 November 2018, accessed March 2025.

[15] Office of National Statistics, 2025, Vital statistics in the UK: births, deaths and marriages, https://www.ons.gov.uk/peoplepopulationandcommunity/populationandmigration/populationestimates/datasets/vitalstatisticspopulationandhealthreferencetables (accessed March 2025).

Probability methods of a type common in reliability engineering were used to determine the likely number of deaths to the end of 2055. A similar method was used to find the number of deaths from radiation-induced solid cancer to the end of 2055. The numbers of deaths in the other survivor cohorts from radiation-induced solid cancer and leukaemia were then calculated by reference to the LSS Core Precursor Cohort and its subcohorts.

It is found that there will likely be 3,118 deaths from radiation-induced solid cancers and leukaemia over all time amongst the 324,000 people who survived the immediate effects of the bombs. Just under one per cent of all survivors can be expected to die from radiation-induced cancers.

Deaths from both radiation-induced disease, both cancer and possible noncancer, will tend to occur late in life, so that the effect on life expectancy will be small. Work carried out under the auspices of the Radiation Effects Research Foundation has found that even survivors who received an average 2,250 mGy, far above the threshold for acute radiation sickness, can expect to live to a median age of 78 years 169 days, nine months more than a baby born in the UK in 2000 could expect to live, and a year and ten months longer than an American baby born in that year.

Industrial pollution may constitute the closest analogy to the long-term effects of the radiation experienced by the survivors of the atomic bombs at Hiroshima and Nagasaki. Survivors will experience a probability of dying that is between 20 and 40 per cent of the probability of dying in the UK from man-made air pollution, according to UK official figures.

The survivors of the atomic bombings have shared in Japan's marked post-war increase in longevity, and their lives will, on average, be decades longer than those of their forebears.

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[16] https://www.bfs.de/EN/topics/ion/radiation-protection/introduction/atomic-bombs/atomic-bombing-protection.html (accessed February 2025).
[17] https://www.mhlw.go.jp/english/database/db-hw/lifetb99_8/index.html (accessed January 2025).
[18] https://www.ipss.go.jp/p-toukei/jmd/00/STATS/bltper_1x1.txt

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Appendix A. Survival probability

A.1 Absolute and conditional survival probabilities

Assume that the lives of a large number $n(0)$ of people are tracked from birth, when their age a is exactly zero, to their deaths, which will happen at different ages, but, for most, will be many years later. Suppose that $n(a_1)$ are alive at age a_1. The survival probability $S(a_1)$ to age a_1 is then the ratio of the number $n(a_1)$ who are alive at age a_1 to the number of live births $n(0)$:

$$S(a_1) = \frac{n(a_1)}{n(0)}. \tag{A.1}$$

There are likely be fewer people still living at a later age $a_2 : a_2 > a_1$, so that $n(a_2) \le n(a_1)$. The corresponding survival probability to age a_2 will be

$$S(a_2) = \frac{n(a_2)}{n(0)}. \tag{A.2}$$

The survival probabilities $S(a_1)$ and $S(a_2)$ may be regarded as absolute probabilities. However, it is often desirable to know the conditional probability $S(a_2|a_1)$ of an individual surviving to age a_2, given that he or she has survived to age a_1. This will be the number $n(a_2)$, alive at age a_2 divided by the number $n(a_1)$ alive at age a_1:

$$S(a_2|a_1) = \frac{n(a_2)}{n(a_1)}. \tag{A.3}$$

But, from eqn (A.1), $n(a_1) = n(0)S(a_1)$, while, from eqn (A.2), $n(a_2) = n(0)S(a_2)$. Substituting these expressions into eqn (A.3) gives the important result:

$$S(a_2|a_1) = \frac{S(a_2)}{S(a_1)}. \tag{A.4}$$

A.2 Interpolation of conditional survival probabilities

Life tables provide demographic information, tabulated at age intervals of one year, from which survival probabilities may be easily calculated. But sometimes it is necessary to find conditional survival probabilities at finer intervals, for which the following interpolation scheme is suitable.

Assume that the probabilities $S(a_1)$ and $S(a_2)$ of surviving from birth to ages a_1 and a_2 are both known. The problem is now to find the unknown survival probability $S(a)$ at age a, where $a_1 < a < a_2$. An approximate solution may be found by assuming that

$$S(a) = S(a_1) + k(a - a_1) \quad \text{for } a_1 \le a \le a_2 \tag{A.5}$$

where k is a constant. Eqn (A.5) gives, at $a = a_2$:

$$S(a_2) = S(a_1) + k(a_2 - a_1). \tag{A.6}$$

[19] Available at https://www.ibiblio.org/hyperwar/Japan/IJA/HB/ (accessed February 2025).

Dividing eqn (A.5) by eqn (A.6) yields:

$$\frac{S(a)-S(a_1)}{S(a_2)-S(a_1)}=\frac{k(a-a_1)}{k(a_2-a_1)}=\frac{a-a_1}{a_2-a_1}. \quad (A.7)$$

Rearranging eqn (A.7) gives:

$$S(a)=S(a_1)+\frac{a-a_1}{a_2-a_1}\left(S(a_2)-S(a_1)\right), \quad (A.8)$$

which, on dividing by $S(a_1)$ yields

$$\frac{S(a)}{S(a_1)}=1+\frac{a-a_1}{a_2-a_1}\left(\frac{S(a_2)}{S(a_1)}-1\right). \quad (A.9)$$

Writing $\Delta a=a-a_1$, and noting that the interval width a_2-a_1 is equal to one year for most life tables, eqn (A.9) becomes:

$$\frac{S(a)}{S(a_1)}=1+\Delta a\left(\frac{S(a_2)}{S(a_1)}-1\right). \quad (A.10)$$

A.3 Relationship between survival probabilities and life expectancy

Being "of age a_n" means that person has an age a'_n that is above exact age a_n and below exact age a_{n+1}. Hence the absolute probability of dying at age a'_n is the probability of surviving to exact age a_n minus the probability of having survived to exact age a_{n+1}:

$$\Pr(A=a'_n)=p_A(a'_n)=S(a_n)-S(a_{n+1}) \quad (A.11)$$

where A is age at death, which will be a random variable. The probability that someone will die at age d'_n, given that he or she has reached exact age a_n, will be the conditional probability:

$$\Pr(A=a'_n|A\ge a_n)=p_A(a'_n|A\ge a_n)= S(a_n|a_n)-S(a_{n+1}|a_n). \quad (A.12)$$

The person may die at age a_n with probability $p_A(a'_n|A\ge a_n)$. Or else he or she may die at some future age $a'_{n+1}, a'_{n+2}, \ldots$, for which the associated probabilities are conditional on having reached exact age a_n are $p_A(a'_{n+1}|A\ge a_n), p_A(a'_{n+2}|A\ge a_n), \ldots$ Thus, by the definition of expected value, the expected age at death for the person who is of age a_n is

$$E(A|A\ge a_n)=\sum_{i=n}^{m} p_A(a'_i|A\ge a_n)a'_i= \sum_{i=n}^{m} p_A(a'_i|A\ge a_n)a_i \quad (A.13)$$

Since the numerical values of a'_i and a_1 are the same. m is chosen, in eqn (A.13), so that the survival probability $S(a_{m+1})\approx 0$, which means that $S(a_{m+1}|a_n)= S(a_{m+1})/S(a_n)\approx 0$ for all a_n. For example, Japanese life tables give full coverage up to age 109.

Hence

$$\begin{aligned}E(A|A\ge a_n)&=a_n\left(S(a_n|a_n)-S(a_{n+1}|a_n)\right)+\\ &a_{n+1}\left(S(a_{n+1}|a_n)-S(a_{n+2}|a_n)\right)+\\ &a_{n+2}\left(S(a_{n+2}|a_n)-S(a_{n+3}|a_n)\right)+\ldots+\\ &(a_m-2)\times\left(S(a_m-2|a_n)-S(a_m-1|a_n)\right)+\\ &(a_m-1)\left(S(a_m-1|a_n)-S(a_m|a_n)\right)+\\ &a_m\left(S(a_m|a_n)-S(a_{m+1}|a_n)\right).\end{aligned} \quad (A.14)$$

But $S(a_n|a_n)=S(a_n)/S(a_n)=1$, while $S(a_{m+1}|a_n)\approx 0$ by assumption. Therefore

$$\begin{aligned}E(A|A\ge a_n)&=a_n+(a_{n+1}-a_n)S(a_{n+1}|a_n)+\\ &(a_{n+2}-a_{n+1})S(a_{n+2}|a_n)+(a_{n+3}-a_{n+2})S(a_{n+3}|a_n)+\\ &\ldots+(a_{m-1}-a_{m-2})S(a_{m-1}|a_n)+(a_m-a_{m-1})S(a_m|a_n).\end{aligned} \quad (A.15)$$

Now $a_{k+1}-a_k=1$ for all k. Hence

$$\begin{aligned}E(A|A\ge a_n)-a_n&=S(a_{n+1}|a_n)+S(a_{n+2}|a_n)+\\ &S(a_{n+3}|a_n)+\ldots+S(a_{m-1}|a_n)+S(a_m|a_n).\end{aligned} \quad (A.16)$$

But $E(A|A\ge a_n)-a_n=E(A-a_n|A\ge a_n)$, which is the expected life-to-come or life expectancy $X(a_n)$ for someone aged a_n. Thus

$$E(A-a_n|A\ge a_n)=X(a_n)=\sum_{i=n+1}^{m} S(a_i|a_n). \quad (A.17)$$

Hence the life expectancy at a general age, a_n, is the sum of the conditional survival probabilities for all physically possible higher ages.

The life expectancy at birth may be found by putting $a_n=0$ into eqn (A.17), so that

$$X(0)=\sum_{i=1}^{m} S(a_i). \quad (A.18)$$

This may be expanded as:

$$\begin{aligned}X(0)&=S(a_1)+S(a_2)+\ldots+S(a_n)+S(a_{n+1})+\ldots+\\ S(a_m)&=S(a_1)+S(a_2)+\ldots+S(a_n)+\\ &S(a_n)\left(\frac{S(a_{n+1})}{S(a_n)}+\frac{S(a_{n+2})}{S(a_n)}+\ldots+\frac{S(a_m)}{S(a_n)}\right)=\\ &\sum_{i=1}^{n} S(a_i)+S(a_n)X(a_n)\end{aligned} \quad (A.19)$$

which splits the problem of finding the life expectancy at birth into two components. This proves useful when modelling the survival function using one expression up to age a_n and a second expression thereafter.

A.4 Improving the accuracy of the calculation of life expectancy at birth

It is possible to obtain a more precise formulation by considering age a to be a continuous rather than a discrete variable, when the equivalent expression to eqn (A.17) may be shown to be

$$E\left(A-a\middle|A>a\right)=X\left(a\right)=\int_{t=a}^{\infty}S\left(t\middle|a\right)\mathrm{d}t. \qquad \text{(A.20)}$$

Applying this to the life tables and integrating using the trapezoidal rule adds a correction of 0.5 to the value given by eqn (A.17):

$$X\left(a_n\right)=\frac{1}{2}+\sum_{i=n+1}^{m}S\left(a_i\middle|a_n\right). \qquad \text{(A.21)}$$

A.4 Relationship between survival probability and probability density for age for a stationary population

Consider a population in a steady state, so that the number of people born each year, $n(0)$, will be constant. Let $n(t)$ (y^{-1}) be the population density at age t, implying that the number of people between ages t and t + dt is $n(t)$dt. The number $n(t)$ may also be regarded as the rate at which people are reaching age t and will be the same as the number of people born t years before, in the interval 0 to dt, multiplied by the probability of surviving to time t, namely $n(0)S(t)$dt:

$$n\left(t\right)\mathrm{d}t=n\left(0\right)S\left(t\right)\mathrm{d}t \qquad \text{(A.22)}$$

Integrating eqn (A.22) between ages 0 and infinity gives:

$$\int_0^{\infty}n\left(t\right)\mathrm{d}t=N=n\left(0\right)\int_0^{\infty}S\left(t\right)\mathrm{d}t \qquad \text{(A.23)}$$

where N is the total number in the population across all ages. But, from eqn (A.20) with $a = 0$,

$$\int_0^{\infty}S\left(t\right)\mathrm{d}t=X\left(0\right) \qquad \text{(A.24)}$$

so that the density of people being born (the rate at which people are being born or birth rate) in a steady-state population is:

$$n\left(0\right)=\frac{N}{X\left(0\right)}. \qquad \text{(A.25)}$$

The fraction of people aged t, that is to say the probability density for age t, may be found by combining (A.25) with (A.22):

$$p\left(t\right)=\frac{n\left(t\right)}{N}=\frac{S\left(t\right)}{X\left(0\right)}. \qquad \text{(A.26)}$$

Appendix B. Using the EYE-Gompertz model to generate life tables for Japan for 1945 and 1946

Unlike the survival probabilities for the years 1947 to 1950, which can be calculated from the official life tables (National Institute of Population and Social Security Research, Japan, 2023), those for 1945 and 1946 need to be estimated from the sparse data available. Useful comments are made in the introduction to the Abridged Life Tables for Japan, 1999 (Ministry of Health, Labour and Welfare, 2025), where it is stated that life expectancy at birth "had been under 50 years till just after the World War II. It was in 1946 when the life expectancies at birth surpassed 50 years for females and in 1947 for males".

This suggests that life expectancy at birth, $X(0)$, in Japan 1946 would have been about 49 years, but a lower figure would have pertained in 1945. Fig. 10 shows a steady increase in life expectancy in Japan between 1947 and 1952, and the life expectancies at birth in 1945 and 1946 have been assigned as 48 and 49 years respectively in a continuation of the trend. The figures chosen for 1945 and 1946 allow for a slow increase in life expectancy in the disordered conditions immediately after the War, followed by a more rapid rise as conditions become more settled and GDP per head increases, reflecting the strong relationship previously established between life expectancy and GDP per head (Thomas, 2017; Thomas and Waddington, 2017).

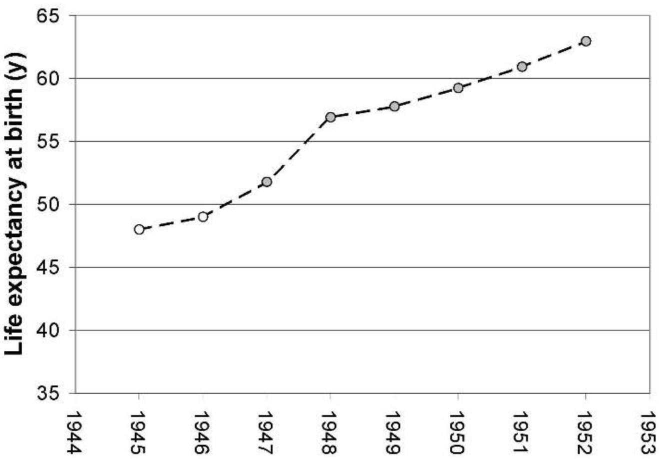

Figure 10. Life expectancy at birth, Japan, 1945 to 1952. Values for 1945 and 1946 are estimates.

B.1 Accounting for Japan's high infant mortality in the 1940s

High mortality up to the age of ten is apparent from the survival probabilities graphed in Fig. 11 for the Japanese population from 1947 to 1950. The sharp drop in survival probability observed over the first few years of life precludes the use of a Gompertz function (Gompertz, 1825) as an accurate predictor of survival over the whole age range. Instead, the survival probabilities up to age 10, from the 1947 life table, were used as the basis for extrapolation to other years using what may be termed the "early years extrapolation" (EYE) model.

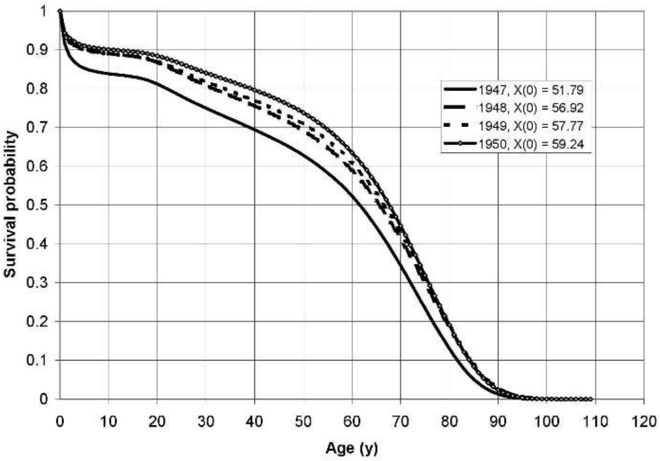

Figure 11. Survival probabilities for Japan's population from 1947 to 1950, based on official life tables.

Plotting the survival probabilities to age 10 for 1948 against the corresponding numbers for 1947 produces an almost linear relationship, as shown in Fig. 12, a phenomenon that is reproduced when the survival probabilities for 1949 and 1950 are plotted against the same basis year. The slopes and intercepts of the linear equations mapping the survival probabilities for ages 0 to 10 in 1947 onto the survival probabilities in other years are given in Table 8.

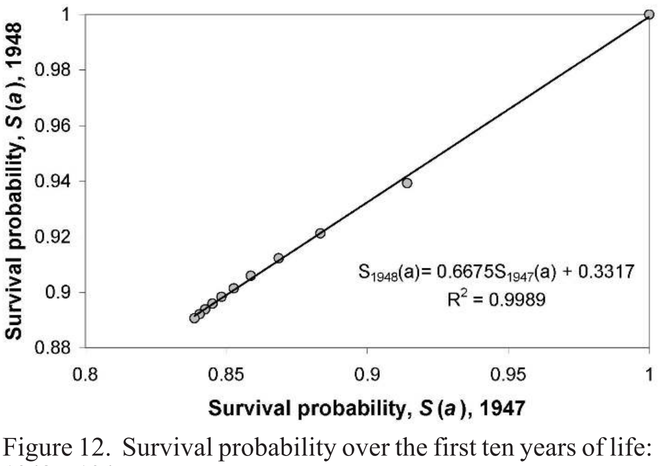

Figure 12. Survival probability over the first ten years of life: 1948 v. 1947.

Table 8. The slopes and intercepts of the linear relationships between survival probabilities for ages 0 to 10 in different years, relative to those of 1947. *R* is the correlation coefficient.

Year	Life expectancy at birth (y)	Life expectancy difference	Slope	Intercept	R^2
1947	51.79	0	1	0	1.0000
1948	56.92	5.13	0.6675	0.3317	0.9989
1949	57.77	5.98	0.6754	0.3235	0.9988
1950	59.25	7.46	0.6047	0.3945	0.9976

The table also lists the life expectancies at birth for the four years and the differences $\Delta X(0)$ between the life expectancy in 1947 and its value in 1948, 1949 and 1950. It is found that both the slope *m* and the intercept *c* of the eqn linking survival probabilities in different years to those of the 1947 base year may be represented, approximately, by the following linear functions of $\Delta X(0)$:

$$\begin{aligned} m\left(\Delta X\left(0\right)\right) &= -0.0543\Delta X\left(0\right)+0.9888 \\ c\left(\Delta X\left(0\right)\right) &= 0.0541\Delta X\left(0\right)+0.0112. \end{aligned} \quad \text{(B.1)}$$

See Fig. 13. The good fit is evident from the high value, > 0.97, of the square of the correlation coefficient in both cases, and from Fig. 14, which compares EYE model predictions with actual life table data.

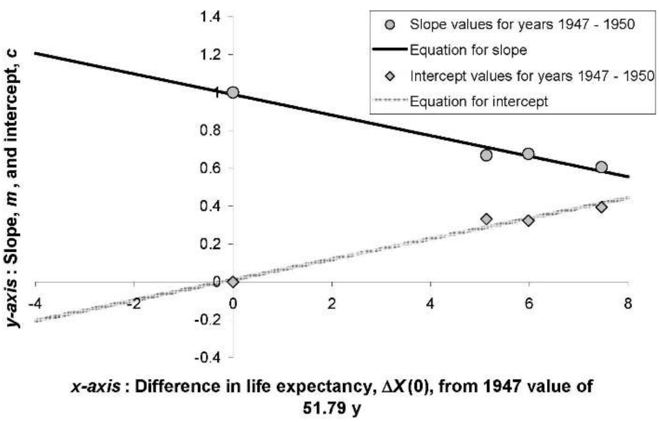

Figure 13. The slope and the intercept of the equations linking survival probabilities for 1948, 1949 and 1950 to the survival probabilities to age 10; both the slope and the intercept are approximately linear in $\Delta X(0)$.

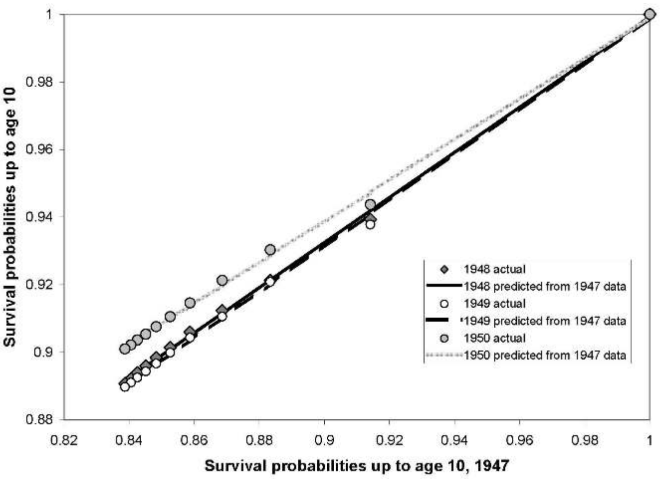

Figure 14. Comparing actual with predicted survival probabilities up to age 10 for years 1948, 1949 and 1950.

The EYE model, where the survival probability $S_y(a)$ at age *a* for year *y* is given by

$$S_y\left(a\right)=m\left(\Delta X\left(0\right)\right)\times S_{1947}\left(a\right)+c\left(\Delta X\left(0\right)\right) \quad 0\le a\le 10 \quad \text{(B.2)}$$

was therefore applied to generate life tables for 1945 and 1946.

B.2 Using Gompertz functions to model the survival probabilities for ages 10 and above

The Gompertz survival function takes the form:

$$S(a) = \exp\left(\frac{b}{g}\left(1 - e^{ga}\right)\right) \quad \text{(B.3)}$$

where b and g are constants, which are found by fitting the curve of eqn (B.3) to empirical data on survival fractions. A Gompertz function, with parameters, $b = 8.43 \times 10^{-5}$ and $g = 8.31 \times 10^{-2}$, was reported by Wilson (1994) to give a close match to the true survival probabilities, conditional on having reached an age of ten years, of 246 million US citizens aged between 10 and 85, based on data from the US census of 1988, when US life expectancy was 74.8 years.[20] A similar procedure was followed here.

Once the survival probabilities, $S(1), S(2), ..., S(10)$, have been calculated using the EYE model, a Gompertz function may be applied to find the conditional survival probabilities, $S(a|10) = S(a)/S(10)$, $a = 10, 11, 12...$ (see Section A.1 of Appendix B) using trial values of b and g:

$$S(a|10) = \exp\left(\frac{b}{g}\left(1 - e^{g(a-10)}\right)\right) \quad \text{(B.4)}$$

where it may be seen, by putting $a = 10$, that $S(10|10) = 1$. The absolute survival probabilities $S(a)$, $a = 10, 11, 12...$ may then be recovered by multiplying by $S(10)$:

$$S(a) = S(10) \times S(a|10) \quad \text{(B.5)}$$

which ensures, *inter alia*, that the survival probability at $a = 10$ that is calculated via the Gompertz formula will match the value produced by the EYE model.

The life expectancy at birth, $X(0)$, may be calculated, using eqn (A.21), as soon as the survival probabilities at all ages up to 109 are available. The Gompertz parameters, b and g, may then be adjusted until the calculated value for at-birth life expectancy matches the value contained in the official life table.

The combined EYE–Gompertz model was used to estimate the shape of the survival curves for 1945 and 1946 under the assumption that the resulting life expectancies at birth should be 48 and 49 respectively. The full set of curves from 1945 to 1950 is shown in Fig. 15.

The EYE-Gompertz model was tested against the survival probabilities for the years 1947 to 1950, as found from the official life tables. Fig. 16 shows the survival probabilities for 1948 calculated using the EYE–Gompertz model compared against life table data, which demonstrates that the model captures the general trend of survival probability with age well if not perfectly.

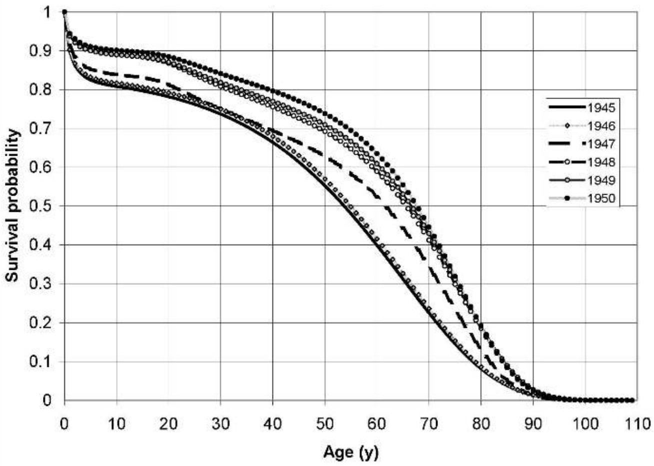

Figure 15. Survival probability curves, 1945 to 1950; the curves for 1945 and 1946 are estimated using the EYE–Gompertz model.

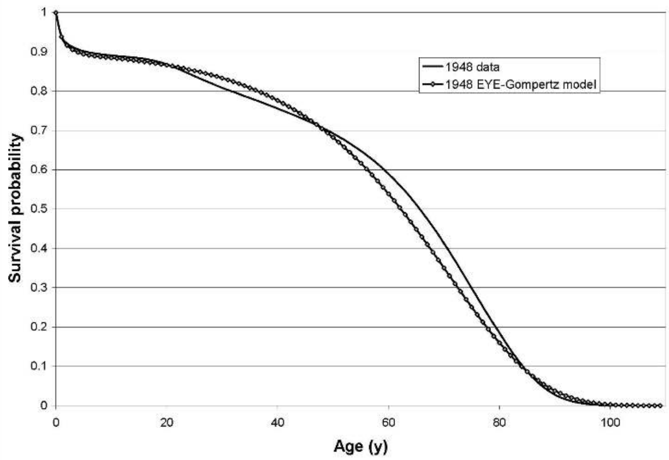

Figure 16. Comparing the survival probabilities from the EYE–Gompertz model with life table data for 1948.

The size of the potential error in the present application is reduced further because the parameter of interest is, as explained in Appendix D, not the survival probability to age a, $S(a)$, but rather the ratio $S(a+1)/S(a)$. Fig. 17 shows that this parameter is captured very well by the EYE–Gompertz model up to an age of about 80. Few people would have lived to this age or above in 1945 or 1946, when the life expectancy at birth, $X(0)$, is known to be under 50. Hence the accuracy of the calculations explained in Appendix D will not be greatly affected by the approximations inherent in the EYE–Gompertz model.

[20] Data Commons (2025), Google, https://datacommons.org/explore#q=life%20expectancy%20of%20US%20in%201988 (accessed January 2025).

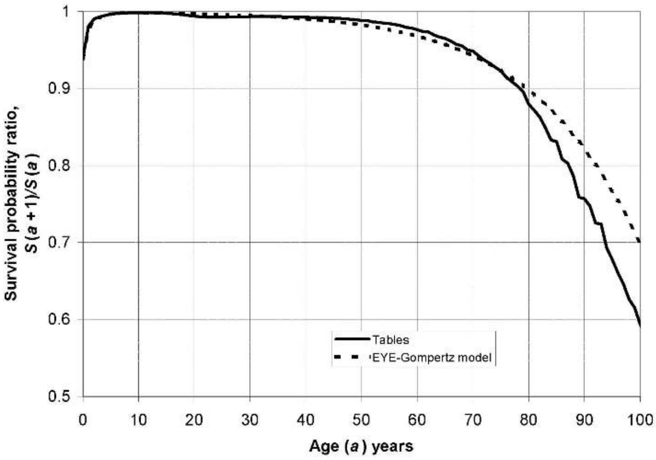

Figure 17. Comparing the survival probability ratio predicted by the EYE–Gompertz model with life table data for 1948.

Appendix C. Determining the age structure of the LSS Core Cohort on 1 October 1950

C.1 Finding, from the age structure at a given date, the corresponding age structure at a later date for a population of survivors

The age structure of the LSS Core Cohort was controlled so that it matched that found in the survivors living in the cities of Hiroshima and Nagasaki in October 1950: "the subjects are representative of the general population of survivors in 1950 with minimal selection and recall bias" (Ozasa et al., 2019). The age distribution of the LSS Core Cohort of 1 October 1950, at the ages its members would have been on the date of the bombings, is given in Table 1, based on Ozasa et al. (2012).

While the people in the LSS Core Cohort all survived from 7 August 1945 to 1 October 1950, they would have formed part of a larger, precursor cohort, some of whom would have died of normal causes in the intervening 5 years. Finding the number and age structure of the LSS Core Precursor Cohort requires the age structure of the LSS Core Cohort on 1 October 1950 to be found. This may be determined using the method now developed.

Consider the people who can be described as of age a on a specified date. Their exact ages t measured in years and fractions of a year, will, in fact, vary between a years and an exact age that is marginally short of $(a + 1)$ years. The total number n_a of people of age a may be found as

$$n_a = \int_a^{a+1} n(t)\,\mathrm{d}t \qquad \text{(C.1)}$$

where $n(t)$ per year is the density of people for exact age t in the interval. (Since the average value of any continuous function, f, on the interval from a to b, is $\bar{f} = (b-a)^{-1}\int_a^b f(x)\,\mathrm{d}x$, and since the length of the interval in this case is one year, n_a is equal to the average density, $\bar{n}$, of $n(t)$ in the interval.)

Fig. 18 shows the population density $p + \Delta$ years after the original date, where p is an integer and Δ is strictly fractional. Referring to the figure, the number n'_p of people aged p at the later date is:

$$n'_p = \int_{t=p}^{p+\Delta} 0\,\mathrm{d}t + \int_{t=p+\Delta}^{p+1} n_0\,\mathrm{d}t = n_0(1-\Delta) \qquad \text{(C.2)}$$

where n_0 is the number of people aged 0 on the original date.

Meanwhile the numbers at later ages, $p+1,\, p+2, \ldots, p+k$ are:

$$\begin{aligned} n'_{p+1} &= \int_{t=p+1}^{p+1+\Delta} n_0\,\mathrm{d}t + \int_{t=p+1+\Delta}^{p+2} n_1\,\mathrm{d}t = n_0\Delta + n_1(1-\Delta) \\ n'_{p+2} &= \int_{t=p+2}^{p+2+\Delta} n_1\,\mathrm{d}t + \int_{t=p+2+\Delta}^{p+3} n_2\,\mathrm{d}t = n_1\Delta + n_2(1-\Delta) \\ &\vdots \\ n'_{p+k} &= \int_{t=p+k}^{p+k+\Delta} n_{k-1}\,\mathrm{d}t + \int_{t=p+k+\Delta}^{p+k+1} n_k\,\mathrm{d}t = n_{k-1}\Delta + n_k(1-\Delta) \end{aligned} \qquad \text{(C.3)}$$

where n_1 is the number of people aged 1 on the original date, n_2 is the number of people aged 2, and so on.

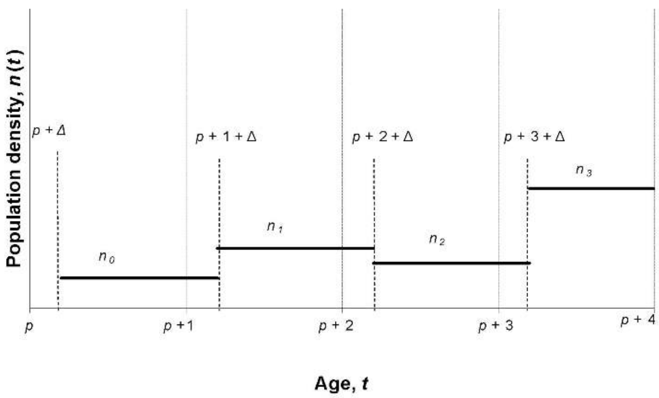

Figure 18. Population density, $n(t)$, $p + \Delta$ years after the original date, where p is an integer and $0 < \Delta < 1$.

By the nature of the human condition, the number of people at some high age, say age $f + 1$, at the original date, will be zero, so that $n_f > 0$ but $n_{f+1} = 0$. It follows from the last line of eqns (C.3) that:

$$\begin{aligned} n'_{p+f+1} &= \int_{t=p+f+1}^{p+f+1+\Delta} n_f\,\mathrm{d}t + \int_{t=p+f+1+\Delta}^{p+f+2} n_{f+1}\,\mathrm{d}t = \\ & n_f\Delta + n_{f+1}(1-\Delta) = n_f\Delta \\ n'_{p+k} &= 0 \text{ for } k > f+1. \end{aligned} \qquad \text{(C.4)}$$

These equations may be applied to the data from Table 1 with p = 5y and Δ = 0.15y. Assuming that the entry for age 50+ contains nonzero data up to age 89, then the age structure for the LSS Core Cohort as of 1 October 1950 is given in Fig. 19.

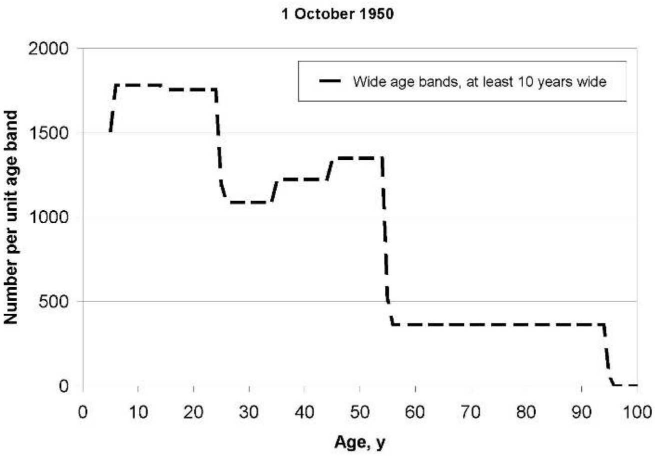

Figure 19. Age structure of the LSS Core Cohort of 86,611 survivors on 1 October 1950 based on wide age bands.

By extension, the age density at an earlier date from the age structure on $p + \Delta$ years later may be found by back-calculation using a rearrangement of the same equations:

$$\begin{aligned} n_0 &= \frac{n'_p}{1-\Delta} \\ n_1 &= \frac{n'_{p+1} - n_0\Delta}{1-\Delta} \\ &\vdots \\ n_k &= \frac{n'_{p+k} - n_{k-1}\Delta}{1-\Delta}. \end{aligned} \quad (C.5)$$

C.2 Estimating the number per unit age band in the LSS Core Cohort on 1 October 1950

Fig. 19 gives the age structure on 1 October 1950 expressed for the age bands 0–9, 10–19, 20–29, 30–39, 40–49, and > 50 years. To estimate the number in unit age bands, viz. at the ages 0, 1, 2, ..., the assumption was made that for the LSS cohort the ratio of the number having an age a to the total number in age band B on 1 October 1950 was proportional to the probability density for age a of Japanese population within the same age band, that is to say:

$$\frac{n_{LSS}(a)}{\sum_{a\in B} n_{LSS}(a)} = \frac{p_{Japan}(a)}{\sum_{a\in B} p_{Japan}(a)} \quad (C.6)$$

where the probability density for Japan as a whole, treated as a steady-state population, is given by applying eqn (A.26):

$$p_{Japan}(a) = \frac{S(t)}{X(0)}. \quad (C.7)$$

Hence

$$\frac{n_{LSS}(a)}{\sum_{a\in B} n_{LSS}(a)} = \frac{S_{Japan}(a)}{\sum_{a\in B} S_{Japan}(a)} \quad (C.8)$$

where the survival probabilities $S_{Japan}(a)$ are found from the 1950 life table for Japan as a whole. See Fig. 20.

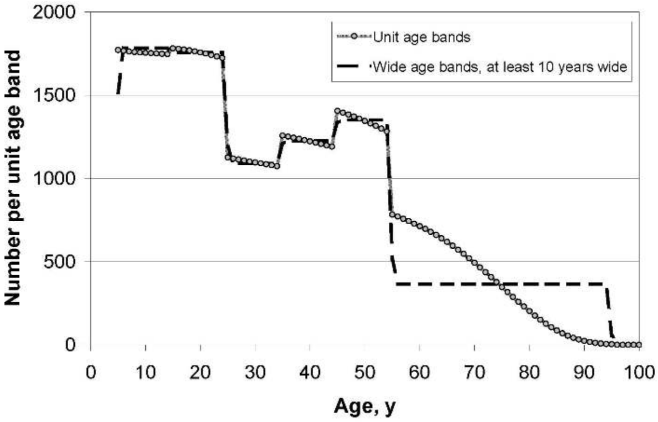

Figure 20. Age structure of the LSS Core Cohort of 86,611 survivors on 1 October 1950: wide age bands and unit age bands compared.

Appendix D. Calculating the age structure of the LSS Core Precursor Cohort and the total number in the LSS Core Precursor Cohort at the time of the bombings

The age structure for 7 August 1945 is found by using the approach set out in Appendix C and extending it to allow for deaths from normal causes.

D.1. Calculating the age density for the LSS Core Precursor Cohort on 7 August 1950 from the age density on 1 October 1950

Five years will have passed between the central date of the bombings and 7 August 1950, which implies, using the terminology introduced in Appendix C, that $p = 5$. Meanwhile 5.15 years will have elapsed between 7 August 1945 and 1 October 1950, implying $p + \Delta = 5.15$, so that $\Delta = 0.15$. Since $p = 5$ is common to both dates, it is simpler to measure dates relative to a start date of 7 August 1950, so that $p = 0$.

The age density for the LSS Core Precursor Cohort is assumed to be available for 1 October 1950 through using the method outlined in Appendix C, whereas the age structure as of 7 August 1950 is unknown. Suppose, however, that the age density on 7 August 1950 were available. Referring to Fig. 18 with $p = 0$, the number of people n' per unit age interval at a time 0.15 years later, namely on 1 October 1950, could, in principle, be found from the earlier numbers per unit age, n, using the following equations, which use survival probabilities to allow for deaths from normal causes:

$$n_0' = \frac{S(\Delta)}{S(0)}\left(\int_{t=0}^{\Delta} 0\mathrm{d}t + \int_{t=\Delta}^{1} n_0 \mathrm{d}t\right) = \frac{S(\Delta)}{S(0)} n_0 (1-\Delta)$$

$$n_1' = \frac{S(1+\Delta)}{S(1)}\left(\int_{t=1}^{1+\Delta} n_0 \mathrm{d}t + \int_{t=1+\Delta}^{2} n_1 \mathrm{d}t\right) = \frac{S(1+\Delta)}{S(1)}\left(n_0\Delta + n_1(1-\Delta)\right)$$

$$\vdots$$

$$n_k' = \frac{S(k+\Delta)}{S(k)}\left(\int_{t=k}^{k+\Delta} n_{k-1} \mathrm{d}t + \int_{t=k+\Delta}^{k+1} n_k \mathrm{d}t\right) = \frac{S(k+\Delta)}{S(k)}\left(n_{k-1}\Delta + n_k(1-\Delta)\right) \text{ for } 0 \le k \le f+1 \tag{D.1}$$

$$n_k' = 0 \text{ for } k > f+1.$$

Here the index parameter f is defined such that $n_f > 0$ but $n_{f+1} = 0$. Meanwhile, survival probabilities $S(k)$ for integer k may be found from the Japanese official life tables (National Institute of Population and Social Security Research, Japan, 2023). The survival probability ratio $S(k+\Delta)/S(k)$ denotes the fraction of those aged k who are still living at an age $k+\Delta$, $0 < \Delta < 1$ that is intermediate between k and $k+1$. This ratio, $S(k+\Delta)/S(k)$ for $k = 0,1,2,...,f+1$, may found by applying eqn (A.10), so that:

$$\frac{S(k+\Delta)}{S(k)} = 1 + \Delta\left(\frac{S(k+1)}{S(k)} - 1\right). \tag{D.2}$$

Inverting eqns (D.1) allows the age structure of the LSS Core Precursor Cohort on 7 August 1950 to be found from the numbers per unit age band for 1 October 1950. The number n_0 of age 0 in the precursor cohort on 7 August 1950 then emerges as:

$$n_0 = \left(\frac{S(\Delta)}{S(0)}\right)^{-1} \frac{n_0'}{1-\Delta} \tag{D.3}$$

and the numbers of people with ages, 1, 2, ... and so on, on 7 August 1950, are found as:

$$n_1 = \frac{\left(\frac{S(1+\Delta)}{S(1)}\right)^{-1} n_1' - n_0\Delta}{1-\Delta}$$

$$\vdots$$

$$n_k = \frac{\left(\frac{S(k+\Delta)}{S(k)}\right)^{-1} n_k' - n_{k-1}\Delta}{1-\Delta} \quad \text{for } 1 \le k < f \tag{D.4}$$

$$n_k = 0 \text{ for } k \ge f.$$

D.2 Using the age density for the LSS Core Precursor Cohort on 7 August 1950 to calculate the age structure an integral number of years before

The eqns (C.2) and (C.3) from Appendix C may be used again as the basis for the analysis by including due allowance for survival.

Proceeding from the earlier date, the age density, $n_0', n_1', ..., n_f', n_{f+1}'$, on 7 August 1950 could be found from the age structure, $n_0, n_1, ..., n_f$, on 7 August 1949, if that were known, by putting $p = 1$ and $\Delta = 0$, so that:

$$n_0' = 0$$

$$n_1' = \frac{S(1)}{S(0)}\int_{t=p}^{p+1} n_0 \mathrm{d}t = \frac{S(1)}{S(0)} n_0$$

$$n_2' = \frac{S(2)}{S(1)}\int_{t=2}^{3} n_1 \mathrm{d}t = \frac{S(2)}{S(1)} n_1$$

$$\vdots$$

$$n_{k+1}' = \begin{cases} \frac{S(k+1)}{S(k)}\int_{t=k+1}^{k+2} n_k \mathrm{d}t = \frac{S(k+1)}{S(k)} n_k & \text{for } 1 \ge k \le f \\ 0 \quad \text{for } k > f \end{cases} \tag{D.5}$$

where the survival probabilities $S(.)$ are found from the life table for 1949.

These eqns may then be inverted to give

$$n_k = \left(\frac{S(k+1)}{S(k)}\right)^{-1} n_{k+1}' \quad \text{for } 0 \le k \le f \tag{D.6}$$

$$n_k = 0 \quad \text{for } k > f.$$

Applying this model successively for the transitions from 1948 to 1949, 1947 to 1948, 1946 to 1947 and, finally, 1945 to 1946 allows the number of people in each of the unit age bands to be found on the central date, 7 August 1945, of the bombings.

The probability density $p_{cities}(k)$ for age k for the two cities on 7 August 1945 may be found as:

$$p_{cities}(k) = \frac{n(k)}{\sum_{k=0}^{109} n(k)} \quad \text{for} \quad k = 0,1,...,109. \tag{D.7}$$

This is compared in Fig. 21 with the population density $p(k)$ for Japan as a whole in 1945, found by applying eqn (A.26) to the 1945 life table. The fall in the early years of life due to high infant mortality is apparent in both curves. A sustained drop in probability density between ages 20 and 40, is also apparent. This would fit with large numbers of men of military age being away from the city, serving in Japan's armed forces.

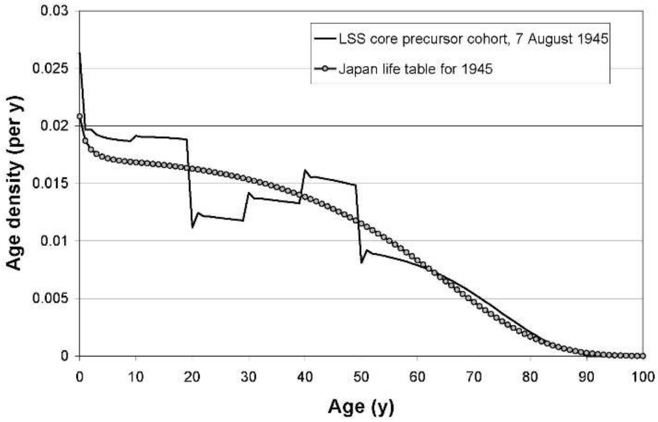

Figure 21. Age density for the LSS Core Precursor Cohort compared with the age density of the Japanese population as a whole in 1945.

D.3 The total number in the LSS Core Precursor Cohort on 7 August 1945

Summing the numbers across all ages reveals that 94,276 people would have been members of the LSS Core Precursor Cohort on 7 August 1945, the central date of the bombings. 7,665 people will have died from normal causes, but 91.89 per cent, or 86,611 people, will still be living on 1 October 1950, the date of the establishment of the LSS Core Cohort.

D.4 Comparing the age structure for the LSS Core Precursor Cohort on 7 August 1945 with that of the LSS Core Cohort on 1 October 1950

The age structure of the LSS Core Precursor Cohort on 7 August 1945 is compared with that of the LSS Core Cohort on 1 October 1950 in Fig. 22. The effect of early childhood mortality has faded after 5 years, but the reduction in probability density for age for those of military age in 1945 persists, as one would expect.

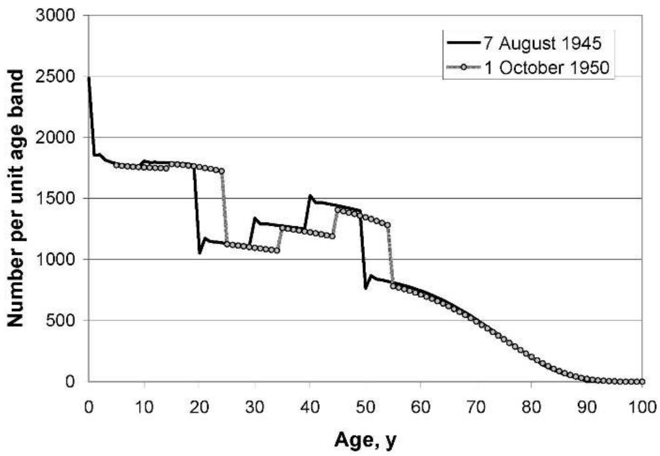

Figure 22. Comparing the age structure of the LSS Core Precursor Cohort of 7 August 1945 with that of the LSS Core Cohort on 1 October 1950.

Appendix E. Properties of demographically similar populations of survivors

E.1 The relationship between population sizes over time

Suppose that there are two populations in the same region and that all the members of each have survived some event, making the populations closed. Let population 1 have size $n_1(t)$ at time t, while population 2 has size $n_2(t)$. The two closed populations may or may not be disjoint.

Let the number of people in population 1 of a given age and sex at time t be denoted $n_{1i}(t)$, and let this number in age-and-sex subpopulation i make up a fraction $a_{1i}(t)$ of population 1 at time t, so that

$$n_{1i}(t) = a_{1i}(t) n_1(t) \quad \text{for } i = 1, 2, ..., N_1. \qquad \text{(E.1)}$$

Here N_1 is the number of age and sex combinations in population 1. For example, if the population consists of males and females at all ages between 0 and 100, then $N_1 = 2 \times 101 = 202$.

Extending the analysis to subpopulation i of population 2 gives

$$n_{2i}(t) = a_{2i} n_2(t) \quad \text{for } i = 1, 2, ..., N_2 \qquad \text{(E.2)}$$

where the subscript i is taken to denote the same age and sex in eqns (E.1) and (E.2).

Now suppose that populations 1 and 2 have similar demographics at some time t_0. Putting $t = t_0$ into eqns (E.1) and (E.2) gives:

$$n_{1i}(t_0) = a_{1i}(t_0) n_1(t_0) \quad \text{for } i = 1, 2, ..., N_1 \qquad \text{(E.3)}$$

$$n_{2i}(t_0) = a_{2i}(t_0) n_2(t_0) \quad \text{for } i = 1, 2, ..., N_2 \qquad \text{(E.4)}$$

while demographic similarity implies that the demographic fractions are equal

$$a_{1i}(t_0) = a_{2i}(t_0) \quad \text{for all } i\text{: } i = 1, 2, ..., N. \qquad \text{(E.5)}$$

Demographic similarity will also mean that there are the same number of subpopulations:

$$N_1 = N_2 = N. \qquad \text{(E.6)}$$

Consider the two subpopulations of populations 1 and 2, which are characterized by age-and-sex, i. Because they live in the same region, the individuals in each demographic subpopulation will, *ceteris paribus*, face the same levels of hazard, so that each such subpopulation will undergo the same relative rate of decline due to normal mortality. Expressed mathematically:

$$\frac{1}{n_{1i}(t_0)} \frac{\mathrm{d}n_{1i}}{\mathrm{d}t} = \frac{1}{n_{2i}(t_0)} \frac{\mathrm{d}n_{2i}}{\mathrm{d}t} \quad i = 1, 2, ..., N. \qquad \text{(E.7)}$$

Formal integration between times t_0 and t gives:

$$\frac{n_{1i}(t) - n_{1i}(t_0)}{n_{1i}(t_0)} = \frac{n_{2i}(t) - n_{2i}(t_0)}{n_{2i}(t_0)} \quad i = 1, 2, ..., N \qquad \text{(E.8)}$$

so that

$$n_{1i}(t)n_{2i}(t_0) - n_{1i}(t_0)n_{2i}(t_0) = n_{1i}(t_0)n_{2i}(t) - n_{1i}(t_0)n_{2i}(t_0) \tag{E.9}$$

which, after simplification, gives the result

$$\frac{n_{1i}(t)}{n_{2i}(t)} = \frac{n_{1i}(t_0)}{n_{2i}(t_0)} \quad i = 1,2,...,N. \tag{E.10}$$

Substituting from eqns (E.3) and (E.4) into eqn (E.10) gives:

$$n_{1i}(t) = \frac{a_{1i}(t_0)n_1(t_0)}{a_{2i}(t_0)n_2(t_0)} n_{2i}(t) \quad i = 1,2,...,N \tag{E.11}$$

or, using eqn (E.5),

$$n_{1i}(t) = \frac{n_1(t_0)}{n_2(t_0)} n_{2i}(t) \quad i = 1,2,...,N. \tag{E.12}$$

Summing both sides of eqn (E.12) yields

$$\sum_{i=1}^{N} n_{1i}(t) = \frac{n_1(t_0)}{n_2(t_0)} \sum_{i=1}^{N} n_{2i}(t). \tag{E.13}$$

The summation on the left hand side will give the total number in population 1 at time, t:

$$\sum_{i=1}^{N} n_{1i}(t) = n_1(t) \tag{E.14}$$

while the summation on the right hand side will give the number in population 2 at time t:

$$\sum_{i=1}^{N} n_{2i}(t) = n_2(t). \tag{E.15}$$

Substituting from eqns (E.14) and (E.15) into eqn (E.13) shows that the ratio of the sizes of two demographically similar populations of survivors will always stay the same as time passes:

$$\frac{n_1(t)}{n_2(t)} = \frac{n_1(t_0)}{n_2(t_0)}. \tag{E.16}$$

Moreover, substituting from eqns (E.1), (E.2) and (E.15) back into eqn (E.12) gives

$$a_{1i}(t)n_1(t) = \frac{a_{1i}(t_0)n_1(t)}{a_{2i}(t_0)n_2(t)} a_{2i}(t)n_2(t) \quad i = 1,2,...,N \tag{E.17}$$

which reduces to

$$a_{1i}(t) = \frac{a_{1i}(t_0)}{a_{2i}(t_0)} a_{2i}(t) \quad i = 1,2,...,N. \tag{E.18}$$

But, by eqn (E.5), $a_{1i}(t_0)/a_{2i}(t_0) = 1$, and so

$$a_{1i}(t) = a_{2i}(t) \qquad i = 1,2,...,N. \tag{E.19}$$

Eqn (E.19) implies that if survivor populations 1 and 2 are matched demographically at any time t_0, they will match demographically at all times.

E.2 The incidence of death from a specified cause in demographically similar populations

Let the function, $f(t)$, be defined by

$$n_1(t) = n_1(t_0)f(t). \tag{E.20}$$

Substituting from eqn (E.20) into eqn (E.16) gives:

$$n_2(t) = n_2(t_0)f(t). \tag{E.21}$$

Integrating eqns (E.20) and (E.21) over a period of duration T years gives:

$$\int_t^{t+T} n_1(t)\mathrm{d}t = n_1(t_0)\int_t^{t+T} f(t)\mathrm{d}t \tag{E.22}$$

and

$$\int_t^{t+T} n_2(t)\mathrm{d}t = n_2(t_0)\int_t^{t+T} f(t)\mathrm{d}t. \tag{E.23}$$

Suppose that m_1 people die from a specified cause in population 1 in the interval of length T. The specific incidence of fatality from this cause per person per year amongst population 1 is then:

$$\frac{m_1}{\int_t^{t+T} n(t)\mathrm{d}t} = \frac{m_1}{n(t_0)\int_t^{t+T} f(t)\mathrm{d}t}. \tag{E.24}$$

Suppose that m_2 people die from the same cause in the same interval in the second population. The specific incidence of fatality (per person per year) is then:

$$\frac{m_2}{\int_t^{t+T} n_2(t)\mathrm{d}t} = \frac{m_2}{n_2(t_0)\int_t^{t+T} f(t)\mathrm{d}t}. \tag{E.25}$$

If the specific incidence of fatality, per person per year, is the same in both populations, then:

$$\frac{m_1}{n_1(t_0)\int_t^{t+T} f(t)\mathrm{d}t} = \frac{m_2}{n_2(t_0)\int_t^{t+T} f(t)\mathrm{d}t}. \tag{E.26}$$

This simplifies to

$$\frac{m_1}{m_2} = \frac{n_1(t_0)}{n_2(t_0)} \tag{E.27}$$

which shows that the ratio of the number of deaths from a specified cause in population 1 to the number of deaths in population 2 in any interval is equal to the ratio of the population sizes at a given time t_0.

Appendix F. Deaths amongst the members of the LSS Core Cohort who were more than 2.5 km from the hypocentre but received a dose higher than 5 mGy

F.1 Deaths from radiation-induced solid cancers

It is deduced in §4.1 that 910 members of the LSS Core Cohort were located more than 2.5 km from the hypocentre but received a colon dose of more than 5 mGy. It is argued there that the average dose experienced by these 910 survivors will be not much more than 5 mGy, so that it is reasonable to assume that they received a radiation dose between 5 mGy and 100 mGy.

Table 9 of Ozasa et al. (2012) shows that 29,961 members of the LSS Core Cohort received a dose between 5 mGy and 100 mGy, and that 49 of those people died from radiation-induced cancer between 1 October 1950 and 31 December 2003. Assuming a uniform distribution for the probability of death in the 53 year interval across the people receiving a dose between 5 and 100 mGy, the fraction of deaths amongst the members located beyond 2.5 km from the hypocentre to the total number will be

$$\frac{910}{29{,}961} = 0.0304 \tag{F.1}$$

and so these 910 people can be expected to have experienced

$$0.0304 \times 49 = 1.5 \text{ deaths.} \tag{F.2}$$

These deaths can be added to the 2 deaths experienced by the 38,509 people who received a colon dose less than 5 mGy and who, by the arguments of §4.1, would have been positioned more than 2.5 km from the hypocentre. Hence the LSS Core Cohort members at this distance or farther away would have experienced 3.5 deaths from radiation-induced solid cancer.

Since the total number of deaths from this cause experienced in the LSS Core Cohort between 1 October 1950 and 31 December 2003 is 527 (Ozasa et al., 2012), the number of radiation-induced solid cancer deaths amongst members closer in than 2.5 km would have been $527 - 3.5 = 523.5$

F.2 Deaths from radiation-induced leukaemia

Table 4 of Richardson et al. (2012) gives the number of radiation-induced leukaemia deaths amongst those members of the LSS Core Cohort receiving a marrow dose of between 5 mGy and 100 mGy between 1 October 1950 and 31 December 2000 as 5.3. Assuming that the number of LSS Core Cohort members located 2.5 km or more from the hypocentre receiving a marrow dose greater than 5 mGy is the same as the number receiving a colon dose higher than 5 mGy, or at least approximately so, then the fraction of eqn (F.1) may be applied to these 5.3 deaths. Hence the number of deaths from radiation-induced leukaemia, between 1 October 1950 and 31 December 2000, experienced by people beyond 2.5 km, is estimated as:

$$0.0304 \times 5.3 = 0.16. \tag{F.3}$$

These deaths can be added to the 0.2 deaths experienced by the 38,509 people who received a marrow dose less than 5 mGy and who, by the arguments of §4.1, would have been positioned over 2.5 km from the hypocentre. Hence the LSS Core Cohort members at this distance or farther away would have experienced 0.36 deaths from radiation-induced leukaemia.

The number of radiation-induced leukaemia deaths amongst those members closer in than 2.5 km would have been $103 - 0.36 = 102.64$.

Appendix G. Estimating the numbers of deaths from radiation-induced leukaemia over all time

G.l. Mathematical method

The number of leukaemias induced by radiation at Hiroshima and Nagasaki is estimated using the methodology applied, successfully, to estimate the size of the outbreak of variant CJD in the UK (Thomas and Newby, 1999, Thomas et al., 2003), as well as radiation-induced childhood thyroid cancers after the Chernobyl reactor accident (Thomas and Zwissler, 2003). The assumption made here is that all radiation-induced cases were initiated at the time of the atomic explosions, taken for simplicity to be the single date, 7 August 1945, as explained in §2.2.

Let the random variable, T, characterize the latency time between exposure on 7 August 1945 and a victim's death from a radiation-induced leukaemia and let there be a theoretical probability density $f_T\left(t \middle| \alpha, \beta\right)$ for T, where the correct choice of the two parameters α and β of the probability distribution will enable the observed results to be reproduced closely. The cumulative probability $F_T(t)$, based on this probability distribution, of death at or before time t is the integral of the probability density:

$$F_T\left(t\right) = \int_{t=t_0}^{t} f_T\left(t \middle| \alpha, \beta\right) \mathrm{d}t. \tag{G.1}$$

The probability of death within any observation interval $t_{i-1} < T \le t_i$, $i = 1, 2, ..., k$ is given by the difference in the cumulative probabilities:

$$F_T\left(t_i\right) - F_T\left(t_{i-1}\right) = \int_{t=t_{i-1}}^{t_i} f_T\left(t \middle| \alpha, \beta\right) \mathrm{d}t. \tag{G.2}$$

Further data points beyond the final time t_k are unknown and may be regarded as, using statistical parlance, "censored".

If, using the chosen probability distribution, $\hat{N}$ is the total number of deaths predicted over all time, viz. as $t \to \infty$, then the number of deaths up to an earlier time t_k will be equal to the product of this number and the probability of dying at or before t_k: $\hat{N}F_T\left(t_k\right)$, in which $F_T(t_k)$ is given by eqn (G.1).

The value of $\hat{N}$ should be such as to enable the probability model to match the number $\sum_{i=1}^{k} n_i$ of deaths observed to the end of the final, kth, observation interval so that

$$\hat{N}F_T\left(t_k\right) = \sum_{i=1}^{k} n_i. \qquad \text{(G.3)}$$

Moreover, the estimated number of deaths $\hat{n}_i$ in each interval will be:

$$\hat{n}_i = \hat{N}\left(F_T\left(t_i\right) - F_T\left(t_{i-1}\right)\right). \qquad \text{(G.4)}$$

Combining eqns (G.3) and (G.4) gives:

$$\frac{\hat{n}_i}{\sum_{i=1}^{k} n_i} = \frac{F_T\left(t_i\right) - F_T\left(t_{i-1}\right)}{F_T\left(t_k\right)} \qquad i = 1,2,...,k. \qquad \text{(G.5)}$$

By eqn (G.5), the ratio $\left(F_T\left(t_i\right) - F_T\left(t_{i-1}\right)\right)/F_T\left(t_k\right)$ will be the estimated number of deaths within interval i, expressed as a fraction of the total deaths observed to time t_k. This fraction is equal to the estimated probability of death within interval i, given that death will occur before time t_k. (Such a conditional probability may be confirmed by substituting from eqns (G.1) and (G.2) into eqn (G.5).)

The probability of n_i deaths occurring within the ith interval, given that there have been $\sum_{i=1}^{k} n_i$ deaths to time t_k is, according to the probability distribution with the chosen values of α and β,

$$\left(\frac{F_T\left(t_i\right) - F_T\left(t_{i-1}\right)}{F_T\left(t_k\right)}\right)^{n_i} \qquad i = 1,2,...,k. \qquad \text{(G.6)}$$

If the n_i, $i = 1,2,...,k$, are assumed to be independent random variables, the likelihood $L(\alpha, \beta)$, given the state of knowledge at time t_k, that the model will predict the number of deaths observed within all the k intervals will then be the product of all the terms referenced in eqn (G.6):

$$L\left(\alpha,\beta\right) = \prod_{i=1}^{k}\left(\frac{F_T\left(t_i\right) - F_T\left(t_{i-1}\right)}{F_T\left(t_k\right)}\right)^{n_i}. \qquad \text{(G.7)}$$

The values of α and β may now be chosen so as to maximize the likelihood of the model matching the observations by causing $L(\alpha, \beta)$ to take its largest value. Moreover, because $\ln(x)$ is monotonically increasing in x, it is possible, as well as often more effective computationally, to find the optimal values of the parameters α and β by maximizing, rather than $L(\alpha, \beta)$, the log-likelihood instead:

$$\ln\left(L\left(\alpha,\beta\right)\right) = \sum_{i=1}^{k} n_i \ln\left(\frac{F_T\left(t_i\right) - F_T\left(t_{i-1}\right)}{F_T\left(t_k\right)}\right). \qquad \text{(G.8)}$$

It may also be necessary, in numerical work, to onstrain the optimization with the requirement that the conditional probabilities sum to unity:

$$\sum_{i=1}^{k}\frac{F_T\left(t_i\right) - F_T\left(t_{i-1}\right)}{F_T\left(t_k\right)} = 1 \quad i = 1,2,...,k. \qquad \text{(G.9)}$$

Substituting eqns (G.1), (G.2) and (G.5) into eqn (G.4) allows the estimated number $\hat{n}_i$ in each interval to be written as:

$$\hat{n}_i = \sum_{i=1}^{k} n_i \frac{\int_{t=t_{i-1}}^{t_i} f_T\left(t|\alpha,\beta\right)\mathrm{d}t}{\int_{t=t_o}^{t_k} f_T\left(t|\alpha,\beta\right)\mathrm{d}t}. \qquad \text{(G.10)}$$

Eqn (G.10) demonstrates that the estimates of the deaths in the intervals depend on the observations and the two distribution parameters α and β. The estimated values $\hat{n}_i$, $i = 1,2,...,k$ may be tested against those observed using the χ^2 test with the appropriate number of degrees of freedom, where

$$\chi^2 = \sum_{i=1}^{k}\frac{\left(\hat{n}_i - n_i\right)^2}{\hat{n}_i}. \qquad \text{(G.11)}$$

One degree of freedom is lost as a result of the full set of n_i appearing both in eqn (G.3) and in maximizing the log-likelihood function given by eqn (G.8). Given that there are k observation intervals, then the number n_{df} of degrees of freedom used in the χ^2 test will be

$$n_{df} = k - 1 - n_p \qquad \text{(G.12)}$$

where n_p is the number of parameters found from the maximizing exercise.

An alternative, and sometimes simpler, procedure is to replace the maximization of the likelihood or log-likelihood function (eqns G.7 and G.8) by choosing α and β so as to minimize the χ^2 statistic given by eqn (G.11). The p-value, which is the probability that the observed level of mismatch between model and observations may be generated by random effects, even though the model is correct, may then be found for the optimal match using either tables or a computer function.

It is normal to regard a p-value of 0.05 or above as acceptable. Provided the optimal match is able to generate a p-value of 0.05 or above, bounding values may be placed on the distribution parameters α and β by establishing the locus of $p = 0.05$. This may be done by increasing α in steps and calculating, at each step, the value of β that produces $p = 0.05$.

Combining eqns (G.1) and (G.3) gives the estimated number of radiation-induced leukaemia deaths as

$$\hat{N} = \frac{\sum_{i=1}^{k} n_i}{\int_{t=t_0}^{t_k} f_T\left(t\middle|\alpha,\beta\right)\mathrm{d}t}. \tag{G.13}$$

As α and β are established at each point on the $p = 0.05$ locus, it follows from eqn (G.12) that the estimated number of radiation-induced leukaemia deaths may be determined at each point also.

G.2 Deaths from radiation-induced leukaemia

G.2.1 For the period from 7 August 1945 to 30 September 1950

It is estimated in §5.2, based on the figures given in Tomonaga (1962), that there would have been 23 deaths from radiation-induced leukaemia prior to 1 October 1950.

G.2.2 For the period from 1 October 1950 to 31 December 2000

Tabulated values of the number of deaths from radiation-induced leukaemia at Hiroshima and Nagasaki for the 5 decades from 1950 to 2000 are provided by Richardson et al. (2009) based on the 310 leukaemia deaths observed in that period among the LSS Core Cohort. Richardson et al. calculate that 103 deaths out of the 310 were attributable to radiation-induced leukaemia.

G.3 The q-value

Table 9 gives the estimated number of radiation-induced leukaemia deaths, divided into six time periods from 7 August 1945 to 31 December 2000.

Table 9. Estimated number of deaths from radiation-induced leukaemia in the LSS Core Precursor Cohort.

From	To	Number of deaths
07-Aug-1945	01-Oct-1950	23
01-Oct-1950	31-Dec-1960	41.8
01-Jan-1961	31-Dec-1970	20.2
01-Jan-1971	31-Dec-1980	13.4
01-Jan-1981	31-Dec-1990	15
01-Jan-1991	31-Dec-2000	12.6
Total		126

It might be argued that the numbers of observed deaths from leukaemia can be regarded as random numbers, with the logical consequence that any function of those numbers, in this case the estimated numbers of radiation-induced leukaemia deaths before 1 October 1950, and subsequently within ten-year intervals, may be regarded as random also. However, the decade by decade figures from Richardson et al. (2009), which cover the period from 1 October 1950 to 31 December 2000, include fractions of a death and are clearly not direct observations. Meanwhile, the 23 deaths from radiation-induced leukaemia prior to 1 October 1950 are based on empirical figures, as shown in §5.2, but, again, the numbers have undergone a number of mathematical processes before the final figure of 23 is found.

Hence, while it is entirely proper to choose to minimize χ^2, which is equivalent to minimizing a weighted sum of squared errors, it is more appropriate to quote q-values rather than p-values. q-values are evaluated using the same procedure as used to find p-values, but the change in nomenclature indicates that the χ^2 statistic is derived from quasi-observations, which have passed through one or more mathematical processes.

The q-value quantifies the goodness of fit in a way that takes due account of both the number of available data points and the number of parameters that need to be estimated. Just as the smallest χ^2 results in the largest p-value, so the lowest χ^2 will lead to the highest q-value. Moreover, the $q = 0.05$ contour will allow reasonable upper and lower bounds to be calculated, taking due account of the number of degrees of freedom. But the resulting q-values should now be regarded as useful indices rather than probabilities.

Appendix H. Estimating the numbers of deaths from radiation-induced solid cancers to the end of 2055 for the LSS Core Precursor Cohort.

A projection is made from the figures contained in Douple et al. (2011) for deaths per year from radiation-induced solid cancers for the LSS Core Cohort, which are themselves based on calculations carried out by Furukawa et al. (2009). Fig. 23 is a digitized version of Fig. 5 of Furukawa et al. (2009). A small corrective multiplier was applied to the digitized figures to ensure that the number of deaths from radiation-induced solid cancers between 1 October 1950 and 31 December 2003 matched the figure, 527, given in Ozasa et al. (2012).

The method based on probability distributions detailed in Appendix G was applied to predicting the numbers of future deaths from radiation-induced solid cancers. It was assumed that there were no deaths from radiation-induced solid cancers prior to 1952. Then the numbers of deaths were found for 22 periods, each roughly 4 years in length. The requirement to find two distribution parameters meant that, from eqn (G.12), the number of degrees of freedom was 19.

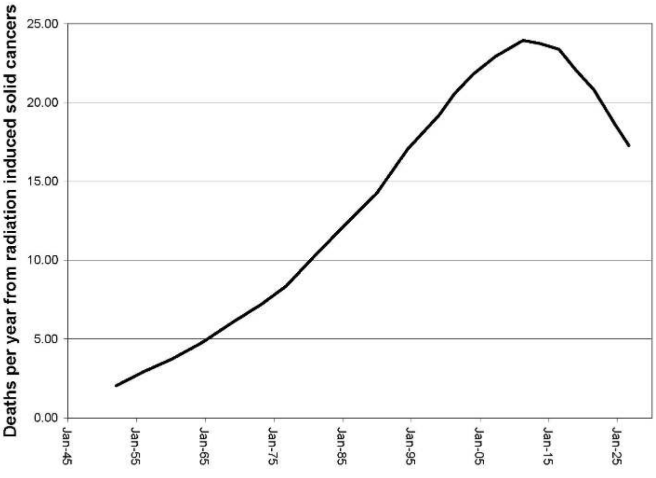

Figure 23. Deaths per year from radiation-induced solid cancers, based on Fig. 5 of Furukawa et al.

The Weibull distribution provided a substantially better fit to the data than the lognormal in this case. Fig. 24 shows the comparison between data and the Weibull model at the optimal match, when

$$\begin{aligned} \alpha &= 2.54 \text{ (dimensionless)} \\ \beta &= 91.98 \text{ years} \\ q &= 0.96 \end{aligned} \tag{H.1}$$

where α is the shape, while β is the characteristic life. The q-value of 0.96 indicates that the match is very good.

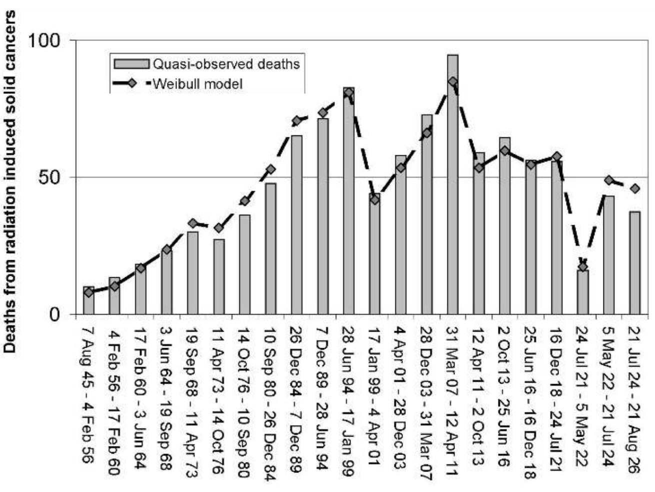

Figure 24. Matching the Weibull distribution to the data for radiation-induced leukaemia deaths.

Fig. 25 shows annual deaths peaking in 2021, ten years after the peak shown in Fig. 23. This has the effect of increasing the number of deaths: constraining the peak to occur at or before 2011 was calculated to lead to 1,402 deaths by the end of 2055, as compared with 1,585 in the unconstrained optimization.

Fig. 26 shows the locus of the q = 0.05 contour, which is closed only at the left hand end, where the minimum number, 1,287, of deaths were projected to

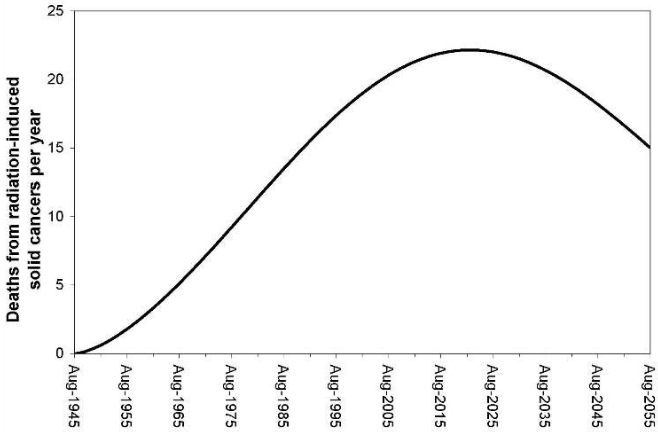

Figure 25. Central estimate of the number of deaths per year from radiation-induced solid cancer.

2055. The locus degenerates into two parallel lines for characteristic lives greater than 10,000 years. The theoretical number of deaths over all time would rise to extremely high values for such a long characteristic life, but truncating the upper date to 2055, when the youngest potential survivor would be 110 years old, reduces the upper bound, consistent with q = 0.05, to 2,132.

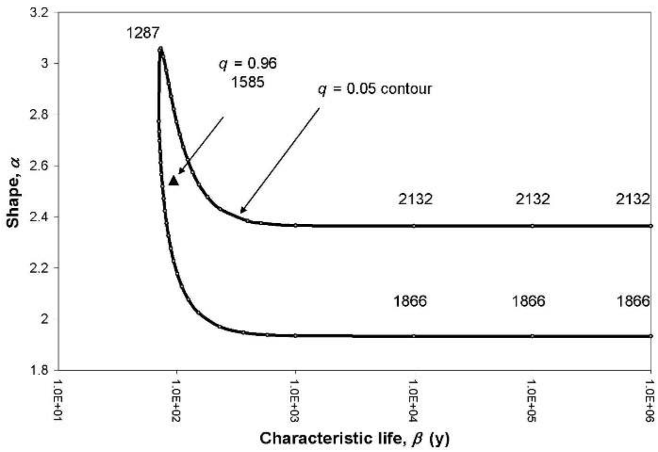

Figure 26. Optimal q-value and contour of q = 0.05 in the plane of shape versus characteristic life; the number of deaths from radiation-induced solid cancer, most likely and bounding values.

Appendix I. Calculation of the number of radiation-induced leukaemia deaths by time period for all cohorts of survivors

I.1 Radiation-induced leukaemia deaths among the civilian survivors resident in one of the two cities on 1 October 1950 but not included in the LSS Core Cohort

I.1.1 Proximal resident civilian survivors who were not part of the LSS Core Cohort

I.1.1.1 Proximal resident civilian survivors not part of the LSS Core Cohort: 1 October 1950 to 31 December 2000

It is calculated in Appendix F that the members of the LSS Core Cohort who were "proximal", that is to say positioned within 2.5 km of the hypocentre, of whom there would have been 47,192 on 1 October 1950, would have incurred 102.64 deaths from radiation-induced leukaemia.

By the arguments of §4.1, there would have been a further 7,130 proximal resident survivors to 1 October 1950 who were omitted from the LSS Core Cohort. The incidence of fatal, radiation-induced leukaemia may be assumed to be the same for these proximal survivors as for the proximal survivors in the LSS Core Cohort. Hence, applying eqn (E.27), and using m in this subsection to signify the number of deaths from radiation-induced leukaemia among the omitted proximal resident survivors:

$$\frac{m}{102.64} = \frac{7130}{47,192} \Rightarrow m = 15.5. \qquad \text{(I.1)}$$

I.1.1.2 Proximal resident civilian survivors not part of the LSS Core Cohort: before 1 October 1950

It is assumed that, just as there was an LSS Core Precursor Cohort that preceded the LSS Core Cohort, so there would have been a precursor cohort to that of the "omitted" proximal resident survivors. Assuming demographical similarity, the ratio of the number of omitted proximal resident survivors on 7 August 1945 to their number on 1 October 1950 will be the same as the ratio of the size of the LSS Core Precursor Cohort on 7 August 1945 to its size on 1 October 1950. On this basis, the number of proximal resident civilians would have been 7,760 in early August 1945.

It is estimated in Section 5.2 that there would have been 23 radiation-induced leukaemia deaths in the LSS Core Precursor Cohort before 1 October 1950. It is reasonable to assume the same distribution of deaths with distance as was found between 1 October 1950 and 31 December 2000. This implies that the number of deaths, μ, from radiation-induced leukaemia experienced by the proximal survivors in the LSS Core Precursor Cohort before 1 October 1950 will obey

$$\frac{\mu}{23} \approx \frac{102.64}{103} \Rightarrow \mu = 23 \times 102.64/103. \qquad \text{(I.2)}$$

Making the assumption that the incidence per person per year of fatal, radiation-induced leukaemia is the same for the omitted proximal resident survivors as for the proximal survivors within the LSS Core Precursor Cohort, and using eqn (E.27), then the number, m, of deaths from radiation-induced leukaemia before 1 October 1950 in the precursor cohort of omitted proximal resident survivors will be:

$$\frac{m}{23 \times 102.64/103} = \frac{7130}{47,192} \Rightarrow m = 3.5. \qquad \text{(I.3)}$$

I.1.1.3 Proximal resident civilian survivors not part of the LSS Core Cohort: from 1 January 2001 to 31 December 2003

It is estimated in §5.3.2 that there were 2.8 deaths from radiation-induced leukaemia amongst the LSS Core Cohort in these two years. Hence, following the reasoning of the past two subsections, the number attributable to the proximal resident cohort not forming part of the LSS Core Cohort is $2.8 \times 102.64/103 \times 7,130/47,192 = 0.4$ deaths from radiation-induced leukaemia.

Thus total number of deaths from radiation-induced leukaemia from 7 August 1945 to 31 December 2003 amongst the cohort of omitted proximal resident survivors and its precursor may be estimated as $3.5 + 15.5 + 0.4 = 19.5$.

I.1.1.4 Proximal resident civilian survivors not part of the LSS Core Cohort: from 7 August 1945 to 31 December 2055

Based on the figure of 161 deaths from radiation-induced leukaemia in the LSS Core Precursor Cohort given in eqn (18), the number μ experienced by the proximal members of the LSS Core Precursor Cohort will obey

$$\frac{\mu}{161} = \frac{102.64}{103} \Rightarrow \mu = 161 \times 102.64/103. \qquad \text{(I.4)}$$

Assuming that the incidence of fatal, radiation-induced leukaemia is the same for the omitted proximal resident survivors as for those proximal survivors within the LSS Core Precursor Cohort , and using m to signify the number of deaths amongst the omitted proximal resident survivors, it is found that, by eqn (E.27),

$$\frac{m}{161 \times 102.64/103} = \frac{7130}{47192} \Rightarrow m = 24.2 \qquad \text{(I.5)}$$

I.1.2 Distal resident civilian survivors who were not part of the LSS Core Cohort, numbering 101,259 on 1 October 1950

I.1.2.1 Distal resident civilian survivors not part of the LSS Core Cohort: from 1 October 1950 to 31 December 2000

It is calculated in Appendix F that the members of the LSS Core Cohort who were "distal", that is to say positioned more than 2.5 km from the hypocentre, of whom there would have been 39,419 on 1 October 1950, would have incurred 0.36 deaths from radiation-induced leukaemia over the period 1 October 1950 to 31 December 2000.

There was, however, a further set of resident distal survivors who were omitted from the LSS Core Cohort. These omitted resident distal survivors numbered 101,259 on 1 October 1950. By analogy with §I.1.1.1, the number of radiation-induced leukaemia deaths amongst these omitted resident distal survivors may be calculated as $0.36 \times 101,259/39,419 = 0.93$.

I.1.2.2 Distal resident civilian survivors not part of the LSS Core Cohort: before 1 October 1950

It is assumed again that, just as there was an LSS Core Precursor Cohort that preceded the LSS Core Cohort, so there would have been a precursor to the cohort of omitted resident distal survivors. Under the assumption of demographic similarity, the ratio of the number of omitted resident distal survivors on 7 August 1945 to their number on 1 October 1950 will be the same as the ratio of the number in the LSS Core Precursor Cohort on 7 August 1945 to their number on 1 October 1950. On this basis, the number of omitted resident distal survivors would have been 110,220 in early August 1945.

There were 23 radiation-induced leukaemia deaths in the LSS Core Precursor Cohort before 1 October 1950 (see §5.2). By analogy with Section I.1.1.2, the number of deaths amongst the distal survivors in the LSS Core Precursor Cohort may be estimated as $23 \times 0.36/103$.

Hence the number of radiation-induced leukaemia deaths amongst the omitted distal resident survivors, who numbered 101,529 on 1 October 1950, plus their precursors, would have been $23 \times 0.36/103 \times 101,259/39,419 = 0.2$.

I.1.2.3 Distal resident civilian survivors not part of the LSS Core Cohort: from 1 January 2001 to 31 December 2003

From §5.3.2, there were 2.8 deaths from radiation-induced leukaemia amongst the LSS Core Cohort in these two years. Hence, by analogy with §I.1.1.3, the number attributable to the omitted distal resident survivors is $2.8 \times 0.36/103 \times 101,259/39,419 = 0.03$.

Thus the estimate of the total number of deaths from radiation-induced leukaemia from 7 August 1945 to 31 December 2003 amongst the omitted distal resident survivors is 0.93 + 0.2 + 0.03 = ~1.2.

I.1.2.4 Distal resident civilian survivors not part of the LSS Core Cohort: from 7 August 1945 to 31 December 2055

Based on the figure of 161 deaths from radiation-induced leukaemia in the LSS Core Precursor Cohort, the number of deaths from this cause, attributable to the omitted distal resident survivors is $161 \times 0.36/103 \times 101,259/39,419 = 1.4$.

I.2 Radiation-induced leukaemia deaths among the civilian survivors who were living away from the two cities after 1 October 1950

There is no information on the location with regard to the hypocentre of these people, who were living away after 1 October 1950 and who numbered $n_{LA}(t_{1950}) = 79,278$ on that date. It is assumed that they had a similar distribution with respect to distance to the "resident" population of civilian survivors, who numbered $n_{RS}(t_{1950}) = 195,000$ on 1 October 1950.

There is no reason to believe that the demographic structure of the population of civilian survivors, who were living away from the two cities by 1 October 1950, will differ from that of the population of resident civilian survivors on that date. The assumption is therefore made that the resident civilian survivors and the moved-away civilian survivors will share the same demographic structure, so that the population of resident civilian survivors becomes the reference population.

Assuming that the incidence of fatal, radiation-induced leukaemia is the same for the resident civilian survivors and the civilian survivors who were living away, then the number of deaths m_{LA} from radiation-induced leukaemia in a specified period among the civilians who were living away, will, by eqn (E.27), obey

$$\frac{m_{LA}}{m_{RS}} = \frac{n_{LA}(t_{1950})}{n_{RS}(t_{1950})} = \frac{79,278}{195,000} \qquad \text{(I.6)}$$

where m_{RS} is the number of deaths amongst the resident civilian survivors in the same period.

The calculated number of deaths in the living-away civilian population are given against time period in Table 5.

I.3 Radiation-induced leukaemia deaths among the military survivors who were living away from Hiroshima before 1 October 1950

In the absence of information on distance from the hypocentre, it is assumed that the geographical spread of these military personnel would have been the same as the resident civilian population of survivors.

The military survivors who were living away from the cities can be expected to have been male and aged between 19 and 44 in 1945 (US War Department, 1944) and hence 24 to 49 on 1 October 1950, which is clearly different from the demographic spread of the civilian population. It is calculated in §3 that there would have been 25,450 such military survivors in early August 1945, and that the number would have dropped to 24,269 on 1 October 1950.

The number of military survivors on 1 October 1950 will be less than 10% of the number of civilian survivors across the two cities, and this will constrain the error introduced by choosing the population of resident civilian survivors as the reference population to a low value.

Assuming that the incidence of death from radiation-induced leukaemia in the population of military survivors is equal to that in the population of resident civilian survivors suggests that deaths from this cause will be about 23 over all time.

Appendix J. Calculation of the number of radiation-induced solid cancer deaths by time period for all cohorts of survivors.

J.1 Radiation-induced solid cancer deaths among the civilian survivors resident in one of the two cities on 1 October 1950 but not included in the LSS Core Cohort

J.1.1 Proximal civilian survivors who were not part of the LSS Core Cohort

J.1.1.1 Proximal civilian survivors not part of the LSS Core Cohort: from 1 October 1950 to 31 December 2003

It is calculated in Appendix F that there would have been 523.5 deaths from radiation-induced solid cancer among the proximal members of the LSS Core Cohort, out of 527 in total between 1 October 1950 and 31 December 2003. There would have been 47,192 proximal survivors in the LSS Core Cohort on 1 October 1950.

It is assumed that the omitted proximal resident survivors, who numbered 7,130 on 1 October 1950, and the proximal survivors in the LSS Core Cohort are demographically similar, and that the incidence of fatal, radiation-induced solid cancer per person per year is the same in the two groups. Denoting the number of deaths from radiation-induced solid cancer by m then, by eqn (E.27),

$$\frac{m}{523.5} = \frac{7130}{47{,}192} \Rightarrow m = 79.1. \qquad \text{(J.1)}$$

J.1.1.2 Proximal civilian survivors not part of the LSS Core Cohort: before 1 October 1950

The long incubation period means that there would have been no deaths from radiation-induced solid cancer before 1 October 1950.

J.1.1.3 Proximal civilian survivors not part of the LSS Core Cohort: from 7 August 1945 to 31 December 2055

By eqn (E.27) the deaths from radiation-induced solid cancer amongst the proximal members of the LSS Core Precursor Cohort will be in proportion to the ratio, 523.5/527, of proximal deaths to all deaths in the LSS Core Cohort between 1 October 1950 and 31 December 2003. Based on the figure of 1585 deaths from radiation-induced solid cancer in the LSS Core Precursor Cohort (Appendix H), the number of deaths μ, experienced over all time by the proximal survivors in the LSS Core Precursor Cohort will obey

$$\frac{\mu}{1585} = \frac{523.5}{527} \Rightarrow \mu = 1585 \times 523.5/527. \qquad \text{(J.2)}$$

It is assumed that the incidence of fatal, radiation-induced leukaemia is the same for the proximal survivors outside the LSS Core Precursor Cohort as for those proximal survivors within it. Then, letting m signify the number of deaths from radiation-induced solid cancer amongst the omitted proximal resident survivors and applying eqn (E.27) gives:

$$\frac{m}{1585 \times 523.5/527} = \frac{7130}{47192} \Rightarrow m = 237.9. \qquad \text{(J.3)}$$

J.1.2 Distal resident survivors (2.5–10 km) who were not part of the LSS Core Cohort, numbering 101,259 on 1 October 1950

J.1.2.1 Distal resident survivors not part of the LSS Core Cohort: from 1 October 1950 to 31 December 2003

From Appendix F, there would have been just 3.5 deaths from radiation-induced solid cancer, out of a total of 527 between 1 October 1950 and 31 December 2003, amongst the distal members of the LSS Core Cohort. There were 39,419 such survivors on 1 October 1950 (see Table 2).

By analogy with §I.1.1.1 of Appendix I, the number of radiation-induced solid cancer deaths amongst these survivors, who numbered 101,259 on 1 October 1950, may be calculated as $3.5 \times 101{,}259/39{,}419 = 9$.

J.1.2.2 Distal resident survivors not part of the LSS Core Cohort: before 1 October 1950

As noted previously, the long incubation period means that there would have been no deaths from radiation-induced solid cancer before 1 October 1950.

J.1.2.3 Distal resident survivors not part of the LSS Core Cohort: from 7 August 1945 to 31 December 2055

Based on the figure of 1,585 deaths from radiation-induced solid cancer in the LSS Core Precursor Cohort over all time, the number of deaths from this cause, attributable to the omitted distal resident survivors is $1{,}585 \times 101{,}259/39{,}419 \times 3.5/527 = 26.9$.

J.2 Radiation-induced solid cancer deaths among the civilian survivors who were living away from the two cities before 1 October 1950

By the arguments of §I.2, the resident population of civilian survivors, who numbered $n_{RS}\left(t_{1950}\right) = 195{,}000$ on 1 October 1950 will constitute the reference population for the civilian survivors who were living away by 1 October 1950.

Assuming that (i) the resident civilian survivors and the living-away civilian survivors share the same demographic structure, and (ii) the incidence of fatal, radiation-induced solid cancers is the same for the resident civilian survivors and the civilian survivors who were living away, then the number of deaths m_{CLA} from radiation-induced solid cancer in a specified period among the civilians who were living away, will, by eqn (E.27), obey

$$\frac{m_{CLA}}{m_{RS}} = \frac{n_{CLA}\left(t_{1950}\right)}{n_{RS}\left(t_{1950}\right)} = \frac{79{,}278}{195{,}000} \qquad \text{(J.4)}$$

where m_{RS} is the number of deaths amongst the resident civilian survivors in the same period.

The numbers of deaths from radiation-induced solid cancers amongst the "living away" civilian survivors are found from applying eqn (J.4). The results are summarized in Table 5.

J.3. Radiation-induced solid cancer deaths among the military survivors who were living away from Hiroshima after 1 October 1950

As noted previously, the error introduced by choosing the population of resident civilian survivors as the reference population for the group of military survivors will be relatively small. It is calculated in §3 that there would have been 25,450 of such military survivors in early August 1945, and that the number would have dropped to 24,269 on 1 October 1950.

Assuming an incidence of death from radiation-induced solid cancer in the population of military survivors that is equal to that in the population of resident civilian survivors, then, by eqn (E.27), the number m_{MLA} of radiation-induced solid cancer deaths amongst the military population that moved away will be given by:

$$\frac{m_{MLA}}{m_{RS}} = \frac{n_{MLA}\left(t_{1950}\right)}{n_{RS}\left(t_{1950}\right)} = \frac{24{,}269}{195{,}000}. \qquad \text{(J.5)}$$

The resulting numbers of deaths from radiation-induced solid cancer by time period are again shown in Table 5.